McGill University
Math 247B: Linear Algebra
Assignment 5: due Friday, March 24, 2000

1. Find a 3×3 matrix A such that

$$A \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} = 2 \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \quad A \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = 2 \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \quad A \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix} = 3 \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}.$$

Find, without computations, the characteristic and minimal polynomials of A. Justify your answers using the theory developed in the course.

2. Let T be the linear operator on $V = \mathbb{R}^{2\times 2}$ defined by

$$T(X) = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix} X.$$

Show that $T^2 - 4T - 5I = 0$ and use this to find a basis for V consisting of eigenvectors of T.

3. If A is the matrix

$$\begin{bmatrix} 9 & -1 & -1 & -1 & 4 \\ -1 & -1 & -1 & 9 & 4 \\ -1 & -1 & 9 & -1 & 4 \\ -1 & 9 & -1 & -1 & 4 \\ 4 & 4 & 4 & 4 & -6 \end{bmatrix},$$

find an invertible matrix P such that $P^{-1}AP$ is a diagonal matrix. Hint: First show that $A^2 = 100I$.

4. If $(x_0, x_1, \ldots, x_n, \ldots)$, $(y_0, y_1, \ldots, y_n, \ldots)$ are two infinite sequences of real numbers satisfying

$$\begin{aligned} x_{n+1} &= x_n + 2y_n \\ y_{n+1} &= -x_n + 4y_n, \end{aligned}$$

find formulae for x_n and y_n given that $x_0 = 1, y_0 = 2$. Find the limiting value of x_n/y_n as n tends to infinity.

5. Find the spectral decomposition of the matrix

$$A = \begin{bmatrix} 5 & 1 \\ 4 & 8 \end{bmatrix}.$$

Compute A^n and find a matrix B such that $B^2 = A$.