McGill University
Math 247B: Linear Algebra
Assignment 2: due Friday, February 4, 2000

1. In each of the following decide whether or not W is a subspace of the vector space V over the field F.

   (a) $V = F^{\mathbb{N}}$, $F = \mathbb{F}_2$, $W = \{\ (x_0, x_1, \dots, x_n, \dots) \in V \mid x_{n+3} = x_{n+2} + x_n \quad \text{for} \quad n \geq 0\}$;

   (b) $V = \mathbb{R}^{\mathbb{R}}$, $F = \mathbb{R}$, $W = \{\ f \in V \mid f(x) = f(x^2) \ \text{ for all } \ x \in \mathbb{R}\}$;

   (c) $V = \mathbb{R}^{[0,1]}$, $F = \mathbb{R}$, $W = \{f \in V \mid f(0)f(1) = 0\}$;

   (d) $V = \mathbb{R}^3$, $F = \mathbb{R}$, $W = \{(x_1, x_2, x_3) \in \mathbb{R}^3 \mid x_1^2 + x_1x_2 + x_2^2 = 0\}$;

   (e) $V = F^{n\times n}$, $W = \{X \in V \mid X^t = -X\}$. $\quad$ (X^t is the transpose of X)

2. Let $(1,1,0,1,1,1), (1,0,1,1,1,0), (1,0,0,1,1,0), (1,1,1,1,1,1)$ be vectors in $\mathbb{F}_2^6$. Find a system of linear equations whose solution space is the subspace of $\mathbb{F}_2^6$ generated by the given vectors.

3. Let

$$\begin{aligned}
U_1 &= \{(x_1, ..., x_4) \in \mathbb{R}^4 : \ x_1 + 2x_2 + x_3 - x_4 = 0\} \\
U_2 &= \{(x_1, ..., x_4) \in \mathbb{R}^4 : \ x_1 = x_2, \ x_3 = x_4\} \\
U_3 &= \mathrm{Span}(((1,1,1,1), \ (1,0,1,0), \ (1,1,0,0))) \\
U_4 &= \mathrm{Span}((1,1,0,1), \ (1,2,2,1)).
\end{aligned}$$

   For each i, j with $1 \leq i < j \leq 4$, find a generating set for $U_i \cap U_j$ and $U_i + U_j$.

4. Let U, V be subspaces of a vector space W.

   (a) Show that $U \cup V$ is a subspace of W if and only if $U \subseteq V$ or $V \subseteq U$.

   (b) If $T : U \times V \to W$ is the mapping defined by $T(u, v) = u + v$, show that T is linear and find the kernel and image of T. Under what conditions is T an isomorphism?