McGill University
Math 240: Discrete Structures 1
Solutions to Class Test

Attempt all questions

1. (a) $\neg p \to (q \to r) \iff (q \to r) \vee p \iff (r \vee \neg q) \vee p \iff \neg q \vee (p \vee r) \iff q \to (p \vee r)$.
   (b) Let $P(x)$ be the statement $x = 0$ and $Q(x)$ the statement $x \neq 0$.

2. (a) $x \in X - \bigcup_{i=1}^{n} A_i \iff x \in X \wedge (\forall i) x \notin A_i \iff (\forall i)(x \in X \wedge x \notin A_i) \iff x \in \bigcap_{i=1}^{n} (X - A_i)$.
   (b) $x \in X - \bigcap_{i=1}^{n} A_i \iff x \in X \wedge (\exists i) x \notin A_i \iff (\exists i)(x \in X \wedge x \notin A_i) \iff x \in \bigcup_{i=1}^{n} (X - A_i)$.

3. (a) R is reflexive since $m + n = m + n \implies (m,n)R(m,n)$ and R is reflexive since

   $$(m,n)R(p,q) \implies m + q = p + n \implies p + n = m + q \implies (p,q)R(m,n).$$

   To show that R is transitive, let $(m,n)R(p,q)$ and $(p,q)R(r,s)$. Then $m + q = p + n$, $p + s = r + q$ which implies $m + s + p + q = r + n + p + q$ on adding these two equations. Cancelling $p + q$ from both sides, we get $m + s = r + n$ which gives $(m,n)R(r,s)$.
   (b) f is independent of the choice of representative in $[(m,n)]$ since $[(m,n)] = [(p,q)] \implies m + q = p + n \implies m - n = p - q$. The mapping f is injective since $f([(m,n)]) = f([(r,s)]) \implies m - n = r - s \implies m + s = r + n \implies (m,n)R(r,s) \implies [(m,n)] = [(r,s)]$. The mapping f is surjective since. for all $n \in \mathbb{N}$, we have $n = f([(n,0)])$, $-n = f([0,n)])$.

4. The proof is by induction on n. For $1 \leq k \leq n$ we have

$$\begin{aligned}\binom{n+1}{k} &= \binom{n}{k-1} + \binom{n}{k} \\ &= \frac{n!}{(k-1)!(n-k+1)!} + \frac{n!}{k!(n-k)!} \\ &= \frac{n!k}{k!(n+1-k)!} + \frac{n!(n+1-k+)}{k!(n+1-k)!} \\ &= \frac{n!(k+n+1-k)}{k!(n+1-k)!} = \frac{(n+1)!}{k!(n+1-k)!}.\end{aligned}$$

   For $k = 0$ and $k = n+1$ we have $\binom{n+1}{0} = 1 = \frac{(n+1)!}{0!(n+1)!}$ and $\binom{n+1}{n+1} = 1 = \frac{(n+1)!}{(n+1)!0!}$.