McGill University
Math 240: Discrete Structures 1
Class Test

Attempt all questions

1. (a) Without using truth tables, show that $\neg p \to (q \to r)$ and $q \to (p \vee r)$ are logically equivalent.

   (b) By means of a counter example, show that $((\forall x)P(x)) \vee ((\forall x)Q(x))$ and $(\forall x)(P(x) \vee Q(x))$ are not logically equivalent.

2. If $A_1, A_2, \ldots, A_n$ are subsets of a set X, we let

$$\bigcup_{i=1}^{n} A_i \quad \text{and} \quad \bigcap_{i=1}^{n} A_i$$

be respectively their union and intersection. Prove that

   (a) $X - \bigcup_{i=1}^{n} A_i = \bigcap_{i=1}^{n} (X - A_i)$;

   (b) $X - \bigcap_{i=1}^{n} A_i = \bigcup_{i=1}^{n} (X - A_i)$.

3. Let $R = \{((m,n),(p,q)) \mid m,n,p,q \in \mathbb{N},\ m+q = p+n\}$.

   (a) Show that R is a equivalence relation on $\mathbb{N} \times \mathbb{N}$.

   (b) If X is the set of equivalence classes prove that $f([(m,n)]) = m - n$ is independent of the choice of representative (m,n) in the equivalence class $[(m,n)]$ and defines a bijective mapping $f : X \to \mathbb{Z}$.

4. For $n, k \in \mathbb{N}$ with $0 \le k \le n$ the natural number $\binom{n}{k}$ is defined inductively by

$$\binom{n}{0} = \binom{n}{n} = 1 \text{ for all } n \text{ and } \binom{n+1}{k} = \binom{n}{k-1} + \binom{n}{k} \text{ for } 1 \le k \le n.$$

Prove that

$$\binom{n}{k} = \frac{n!}{k!(n-k)!} = \frac{n(n-1)\cdots(n-k+1)}{k!}.$$