McGill University
Math 240: Discrete Structures 1
Sample Test

Class Test Wednesday October 26

1. For each of the following statements

$$p \to (q \to r), \quad (p \to q) \to r)$$

find logically equivalent statement involving only the symbols $p, q, r, \neg p, \neg q, \neg r, \wedge, \vee$. Are the two given statements logically equivalent? Justify your assertions.

2. Prove or disprove the following statements:

   (a) For all sets A, B, C, D

$$(A - B) - (C - D) = (A - (B \cup C)) \cup ((A \cap D) - B).$$

   (b) For all sets A, B, C, D

$$(A - B) - (C - D) = (A - C) - (B - D).$$

3. (a) Explain carefully what it means for a mapping $f :\to Y$ to be injective, surjective, bijective.
   (b) In each of the following cases, give an example of a function $f : \mathbb{N} \to \mathbb{N}$ which is
      (i) injective but not surjective,
      (ii) surjective but not injective,
      (iii) bijective and not the identity mapping,
      (iv) neither injective nor surjective.

4. If $(a_n)_{n \geq 0}$ is defined inductively by

$$a_0 = 1, a_1 = 1, a_n = a_{n-1} + 2a_{n-2} \text{ for } n \geq 2,$$

prove by induction that $a_n = (2^{n+1} + (-1)^n)/3$ for all n.