

McGill University
Math 240: Discrete Structures 1
Assignment 6: due Friday, December 2, 2005

Reading: Text 6.6: Applications of Inclusion-Exclusion, 8.1: Introduction to Graphs, 8.2: Graph Terminology, 8.3: Representing Graphs and Graph Isomorphism, 8.4: Connectivity, 8.5: Euler and Hamilton Paths, 9.1: Introduction to Trees, 9.4: Spanning Trees

Questions:

1. How many solutions $(x_1, x_2, x_3, x_4) \in \mathbb{N}^4$ are there to the equation

$$x_1 + x_2 + x_3 + x_4 = 15$$

if $x_1 \leq 3$, $x_2 \leq 4$, $x_3 \leq 3$ and $x_4 \geq 4$.

2. Find, up to isomorphism, all simple graphs with 4 vertices. The graphs you find should be pairwise non-isomorphic. Justify your assertions.
3. Draw the graphs with the following adjacency matrices and determine which pairs of graphs are isomorphic.

$$\begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \end{bmatrix}, \quad \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 \end{bmatrix}, \quad \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}.$$

4. For each of the graphs in question 3, find a spanning subtree.