McGill University
Math 240: Discrete Structures 1
Assignment 2: due Wednesday, September 28, 2005

Reading: Text 1.8: Functions, 7.1: Relations, 7.5: Equivalence Relations.

Questions:

1. For a relation on a set A show that the properties of reflexivity, symmetry and transitivity are independent by finding
   (a) a relation R on A which is reflexive and symmetric but not transitive;
   (b) a relation R on A which is reflexive and transitive but not symmetric;
   (c) a relation R on A which is symmetric and transitive but not reflexive.

2. Let $f : X \to Y$ be a mapping from the set X to the set Y.
   (a) If A, B are subsets of X prove that if f is injective then $f(A \cap B) = f(A) \cap f(B)$ and $f(A - B) = f(A) - f(B)$.
   (b) If A, B are subsets of Y prove that $f^{-1}(A \cap B) = f^{-1}(A) \cap f^{-1}(B)$ and $f^{-1}(A - B) = f^{-1}(A) - f^{-1}(B)$.

3. Let X be a set and let 2^X denote the set of all functions f with domain X and range a subset of $2 = \{0, 1\}$. For any subset A of X let $\chi_A \in 2^X$ be defined by
$$\chi_A(x) = \begin{cases} 1 & \text{if } x \in A, \\ 0 & \text{if } x \notin A. \end{cases}$$
   The function χ_A is called the characteristic function of the subset A. Show that the mapping $A \mapsto \chi_A$ of $P(X)$ to 2^X is bijective.

4. Let $f : X \to Y$ and $g : Y \to Z$ be mappings. Show that
   (a) $g \circ f : X \to Z$ injective $\implies$ $f : X \to Y$ injective;
   (b) $g \circ f : X \to Z$ surjective $\implies$ $g : Y \to Z$ surjective;
   (c) if $Z = X$ and $g \circ f = 1_X$ and $f \circ g = 1_Y$ then $f : X \to Y$ and $g : Y \to X$ are bijective and $g = f^{-1}$ and $f = g^{-1}$.