McGill University Math 236: Algebra 2 Assignment 2: due Friday, February 3, 2006

- 1. Let V be a vector space over a field F and let $v_1, v_2, v_3, v_4, w \in V$.
 - (a) Prove that if (v_1, v_2, v_3, v_4) spans V then so does $(v_1 v_2, v_2 v_3, v_3 v_4, v_4)$.
 - (b) Prove that if (v_1, v_2, v_3, v_4) is linearly independent then so is $(v_1 v_2, v_2 v_3, v_3 v_4, v_4)$.
 - (c) Show that $(v_1 v_2, v_2 v_3, v_3 v_4, v_4 v_1)$ is linearly dependent.
 - (d) If (v_1, v_2, v_3, v_4) is linearly independent and $(v_1 + w, v_2 + w, v_3 + w, v_4 + w)$ is linearly dependent, show that $w \in \text{Span}(v_1, v_2, v_3, v_4)$
- 2. Determine whether the given sequence of real vectors are linearly independent or not. In the case of linear dependence find a non-trivial dependence relation.
 - (a) $(1,1,1,1), (1,1,2,2), (2,2,1,1), (1,1,3,3), (4,4,3,3) \in \mathbb{R}^4$.
 - (b) $f(x) = \sin(x+1), g(x) = \sin(x+2), h(x) = \sin(x+3) \in \mathbb{R}^{\mathbb{R}}$
 - (c) $f(x) = e^x, g(x) = xe^{2x}, h(x) = x^2e^{3x} \in \mathbb{R}^{\mathbb{R}}$
 - (d) $(1, 2, 3, \dots, n, \dots), (1, 2^2, 3^2, \dots, n^2, \dots), (1, 2^3, 3^3, \dots, n^3, \dots) \in \mathbb{R}^{\infty}$
- 3. (a) Find a basis for the subspace of \mathbb{R}^4 spanned by the vectors in 2(a) and complete this basis to a basis of \mathbb{R}^4 .
 - (b) Show that the subspace in 3(a) is equal to Span((3,3,7,7),(7,7,3,3)).
- 4. Let U = Span((2,1,1,3), (1,2,2,1), (3,2,1,5)), V = Span((1,4,3,5), (1,1,2,2), (4,3,1,7)).Find bases for U + V and $U \cap V$.
- 5. Let U be the subspace of \mathbb{R}^{∞} defined by

$$U = \{ x = (x_1, x_2, \dots, x_n, \dots) \in \mathbb{R}^{\infty} \mid x_{n+4} = 5x_{n+2} - 4x_n (n \ge 1) \}.$$

Prove that the infinite sequences x, y, z, w defined by

$$x_n = 1$$
, $y_n = (-1)^{n-1}$, $z_n = 2^{n-1}$, $w_n = (-2)^{n-1}$

form a basis for U. You may assume as known that $\dim(U) = 4$. Use this to find a formula for the *n*-th term of the infinite sequence $u \in U$ where $u_1 = 1, u_2 = 2, u_3 = 3, u_4 = 4$.