McGill University
Math 223 B: Linear Algebra
Midterm exam: Wednesday March 3, 1999

Attempt all Questions.
All questions are of equal value.
No Calculators Allowed.

1. Let V be the vector space of infinite sequences of real numbers. Which of the following subsets W of V are subspaces of V? Justify your assertions.

   (a) $W = \{x = (x_0, x_1, x_2, \ldots, x_n \ldots) \in \mathrm{V} \mid x_{n+2} = 2x_{n+1} + x_n\}$

   (b) $W = \{x = (x_0, x_1, x_2, \ldots, x_n \ldots) \in \mathrm{V} \mid x_{n+2} = x_{n+1} + x_n^2\}$

2. Let F be the vector space of real-valued functions on the real line. Which of the following sets of functions are linearly independent and which are linearly dependent? Justify your assertions.

   (a) $\{1, xe^x, x^2\}$;

   (b) $\{e^{x+1}, e^{x+3}, e^{x+5}\}$.

3. Let V be the vector space of 2×2 matrices and let W be the subspace consisting of those matrics $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ with $a + d = b - c = 0$.

   (a) Find a basis for W. What is the dimension of W?

   (b) Complete the basis found in (a) to a basis for V.