McGill University
Math 223B: Linear Algebra
Assignment 6

Not to be handed in. Nevertheless it is strongly recommended to do these exercises.

1. Let P_2 be the vector space of polynomials of degree at most 2. Let $T : P_2 \to P_2$ be the linear operator
$$f(x) \mapsto (1-x)f'(x).$$
   a) Find the matrix $M_B(T)$ of T with respect to the ordered basis $B = \{1, x, x^2\}$ of P_2.
   b) Use part a) to find the eigenvalues and eigenvectors, the kernel and the image of T.

2. Which of the following are inner products on $\mathbb{R}^2$? Justify your answers.
   a) $\langle (x_1, x_2), (y_1, y_2)\rangle = x_1y_1 + 2x_1y_2 + x_2y_2$,
   b) $\langle (x_1, x_2), (y_1, y_2)\rangle = x_1y_2 + x_2y_1$,
   c) $\langle (x_1, x_2), (y_1, y_2)\rangle = 2x_1y_1 - x_1y_2 - x_2y_1 + x_2y_2$,
   d) $\langle (x_1, x_2), (y_1, y_2)\rangle = x_1y_1 + 2x_1y_2 + 2x_2y_1 + x_2y_2$.

3. a) Show that
$$(x_1, x_2, x_3), (y_1, y_2, y_3)\rangle = x_1y_1 + x_1y_2 + x_2y_1 + 6x_2y_2 + 2x_2y_3 + 2x_3y_2 + x_3y_3$$
   is an inner product on $\mathbb{R}^3$.
   b) Find the orthogonal projection (with respect to the inner product in a)) of $Z = (-2, 5, -2)$ on $U = span\{(1, 0, -1), (0, 1, 3)\}$.

4. Let V be the vector space of all polynomials with inner product
$$\langle f(x), g(x)\rangle = \int_0^1 f(x)g(x)dx.$$
   Find a basis of $U = span\{1, x^2, x^4\}$ which is orthogonal with respect to $\langle\ ,\ \rangle$.