McGill University
Math 223B: Linear Algebra
Assignment 4: due Wednesday, March 17, 1999

1. Find the eigenvalues and eigenspaces of the matrix

$$A = \begin{pmatrix} 2 & -2 & 1 & 1 \\ 0 & 3 & 0 & 0 \\ -1 & -2 & 4 & 1 \\ 1 & -2 & -2 & 1 \end{pmatrix}.$$

2. Let V be the vector space of all polynomials and let $\phi : V \to V$ be the linear operator

$$f(x) \mapsto xf'(x).$$

Is ϕ one-to-one? Is ϕ onto? Find all eigenvalues and eigenspaces of ϕ.

3. Decide whether the matrices A and B are similar:

a)

$$A = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix}$$

b)

$$A = \begin{pmatrix} 2 & 3 \\ 4 & -1 \end{pmatrix}, \quad B = \begin{pmatrix} 2 & -3 \\ 4 & -1 \end{pmatrix}$$

c)

$$A = \begin{pmatrix} 5 & 1 \\ 0 & 4 \end{pmatrix}, \quad B = \begin{pmatrix} 5 & 0 \\ 1 & 4 \end{pmatrix}$$

4. Diagonalize the matrix

$$A = \begin{pmatrix} -7 & -9 \\ 6 & 8 \end{pmatrix}$$

and use this to calculate A^n for every $n \in \mathbb{N}$.

5. Let

$$U = \operatorname{span}\left\{ \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} -3 \\ 1 \\ 0 \\ 6 \end{pmatrix} \right\} \subseteq \mathbb{R}^4.$$

Find an orthonormal basis for U.