McGill University
Math 223B: Linear Algebra
Assignment 2: due Wednesday, February 3, 1999

1. Which of the following are subspaces of the vector space V of differentiable functions on $\mathbb{R}$?

   a) $\{f \in V \mid f(2) = f(3) - 4\}$,
   b) $\{f \in V \mid f(2) = f(3) - f(4)\}$,
   c) $\{f \in V \mid f(2) \leq f(3)\}$,
   d) $\{f \in V \mid f'(2) = 0\}$.

   Justify your answers.

2. Show that the set of vectors

$$\left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \\ -1 \end{pmatrix}, \begin{pmatrix} 0 \\ -5 \\ 5 \\ -8 \end{pmatrix}, \begin{pmatrix} 3 \\ 3 \\ 3 \\ 3 \end{pmatrix}, \begin{pmatrix} 7 \\ 0 \\ -2 \\ 4 \end{pmatrix} \right\}$$

   in $\mathbb{R}^4$ is linearly independent.

3. Consider the set $M = \{(2,3,4),\ (8,-9,7),\ (0,7,3)\}$ in $\mathbb{R}^3$. Give two different representations of $(4,20,14)$ as a linear combination of vectors in M. Does the vector $(4,8,16)$ lie in the span of M? Is M linearly independent?

4. Show that $\{5x^4,\ \sin(3x),\ e^{-x}\}$ is linearly independent in the vector space of differentiable functions on $\mathbb{R}$.

5. a) Show that the span of $M = \{x^3,\ x^2,\ x^3 - 5x^2,\ 4\}$ is the vector space

$$W = \{p(x) \mid p(x) \text{ is a polynomial of degree } \leq 3 \text{ with } p'(0) = 0\}.$$

   Is M a basis of W? What is the dimension of W?

   b) Let V be the vector space of all 3×3 matrices A with $A^\top = -A$. Find a basis of V. What is the dimension of V?