McGill University Math 223B: Linear Algebra Assignment 2: due Wednesday, February 3, 1999

1. Which of the following are subspaces of the vector space V of differentiable functions on \mathbb{R} ?

a)
$$\{f \in V \mid f(2) = f(3) - 4\}$$

b) $\{f \in V \mid f(2) = f(3) - f(4)\},\$

c)
$$\{f \in V \mid f(2) \le f(3)\},\$$

d) $\{f \in V \mid f'(2) = 0\}.$

Justify your answers.

2. Show that the set of vectors

$$\left\{ \left(\begin{array}{c}1\\0\\-1\end{array}\right), \left(\begin{array}{c}0\\-5\\5\\-8\end{array}\right), \left(\begin{array}{c}3\\3\\3\\3\end{array}\right), \left(\begin{array}{c}7\\0\\-2\\4\end{array}\right) \right\}$$

in \mathbb{R}^4 is linearly independent.

- 3. Consider the set $M = \{(2,3,4), (8,-9,7), (0,7,3)\}$ in \mathbb{R}^3 . Give two different representations of (4,20,14) as a linear combination of vectors in M. Does the vector (4,8,16) lie in the span of M? Is M linearly independent?
- 4. Show that $\{5x^4, \sin(3x), e^{-x}\}$ is linearly independent in the vector space of differentiable functions on \mathbb{R} .
- 5. a) Show that the span of $M = \{x^3, x^2, x^3 5x^2, 4\}$ is the vector space

 $W = \{ p(x) \mid p(x) \text{ is a polynomial of degree } \le 3 \text{ with } p'(0) = 0 \}.$

Is M a basis of W? What is the dimension of W?

b) Let V be the vector space of all 3×3 matrices A with $A^{\top} = -A$. Find a basis of V. What is the dimension of V?