McGill University
Math 223B: Linear Algebra
Assignment 1: due Friday, January 22 1999

1. Solve the system

$$\begin{aligned} 2x_1 - 3x_2 + 6x_3 + 2x_4 - 5x_5 &= 2 \\ 2x_1 - 2x_2 + 2x_3 + 3x_4 - 5x_5 &= 0 \\ -2x_1 + 2x_2 - 2x_3 - 4x_4 + 8x_5 &= 2 \\ 5x_1 - 6x_2 + 9x_3 + 7x_4 - 14x_5 &= 1 \end{aligned}$$

by finding the row reduced echelon form of its augmented matrix. Clearly indicate the elementary operations that you perform in your row reduction and write the solutions in parametric form.

2. If A is the coefficient matrix of the system of equations

$$\begin{aligned} 9x_1 - x_2 - x_3 - x_4 + 4x_5 &= 1 \\ -x_1 - x_2 - x_3 + 9x_4 + 4x_5 &= 2 \\ -x_1 - x_2 + 9x_3 - x_4 + 4x_5 &= 3 \\ -x_1 + 9x_2 - x_3 - x_4 + 4x_5 &= 4 \\ 4x_1 + 4x_2 + 4x_3 + 4x_4 - 6x_5 &= 5, \end{aligned}$$

solve this system by showing that $A^2 = 100I$ and using this to find the inverse of A.

3. Find an LU-factorization of the matrix

$$A = \begin{bmatrix} 2 & 6 & -2 & 0 & 2 \\ 3 & 9 & -3 & 3 & 1 \\ -1 & -3 & 1 & -3 & 1 \end{bmatrix}$$

and a PLU-factorization of the matrix

$$B = \begin{bmatrix} 0 & -1 & 2 & 1 \\ -1 & 1 & 3 & 1 \\ 1 & -1 & -3 & 6 \\ 2 & -2 & -4 & 1 \end{bmatrix}.$$

Use your factorization of B to find $\det(B)$.

4. Solve

$$\begin{aligned} (1+i)x + 3y &= i \\ x + (1-2i)y - iz &= 0 \\ 2x - 4iy + (1-i)z &= 1 \end{aligned}$$

over the complex numbers by (i) Gaussian eimination and (ii) Cramer's Rule.

5. Let d_n be the determinant of the $n \times n$ matrix A_n whose diagonal elements are all equal to 3, the entry immediately above any diagonal entry is 2, the entry immediately below any diagonal entry is 1 and all other entries equal to zero. Using induction, prove, that $d_n = 2^{n+1} - 1$. **Hint**: Using row and column expansions, show that $d_n = 3d_{n-1} - 2d_{n-2}$ for $n \geq 3$.