1. For each of the following series find (i) the radius of convergence and (ii) what happens at the endpoints of the interval of convergence.

$$(a)\ \sum_{n=1}^{\infty} \frac{(-1)^n x^{2n}}{n4^n}, \qquad (b)\ \sum_{n=0}^{\infty} \frac{x^{3n}}{64^n \cdot \sqrt{n+1}}.$$

2. Let

$$F(x) = \int_0^x \frac{\sin(t^2)}{t} dt.$$

   (a) Find the Maclaurin series of $F(x)$.

   (b) Find the radius of convergence of the series in (a).

   (c) Evaluate $F(0.2)$ correct to 6 decimals.

3. (a) Find the unit tangent, principal normal and curvature of the curve with parametric equations $x = t^3/3,\ y = 2t,\ z = 2/t$ at any point of the curve where $t > 0$.

   (b) Find (i) the equation of the tangent line to the curve in (a) at the point where $t = 1$ and (ii) the length of that part of this curve which is between the planes $z = 1$ and $z = 2$.

4. If $u = f(r)$, where f is differentiable and $r = \sqrt{x^2 + y^2 + z^2}$, show that, for $(x, y, z) \neq (0, 0, 0)$,

$$(a)\ (\frac{du}{dr})^2 = (\frac{\partial u}{\partial x})^2 + (\frac{\partial u}{\partial y})^2 + (\frac{\partial u}{\partial z})^2 \quad \text{and} \quad (b)\ \nabla u = \frac{1}{r}\frac{du}{dr}(x\vec{i} + y\vec{j} + z\vec{k}).$$

5. (a) Find the equation of the tangent plane and normal line to the surface $z = 3xe^y - x^3 - e^{3y}$ at the point $(0, 0, -1)$.

   (b) Show that the function $f(x, y) = 3xe^y - x^3 - e^{3y}$ has a unique critical point and that this critical point is a local maximum but not a global maximum.

6. Suppose that $T(x, y, z) = x^3y + y^3z + z^3x$ is the temperature at the point (x, y, z) in 3-space.

   (a) Calculate the directional derivative of T at the point $P(2, -1, 0)$ in the direction from P to the point $Q(1, 1, 2)$.

   (b) A mosquito is flying through space with constant speed 5 in the direction of increasing temperature. If the mosquito's direction of flight at any given point is always normal (perpendicular) to the level surface of $f(x, y, z) = 2x^2 + 3y^2 + z^2$ passing through this point, find the rate of change of temperature experienced by the mosquito when it is at the point $(2, -1, 0)$. Hint: $\frac{dT}{dt} = \nabla T \cdot \vec{v}$.

7. Reverse the order of integration and then evaluate

$$\int_0^2 \Big(\int_0^{x^2/2} \frac{x}{(1 + x^2 + y^2)^2} dy \Big) dx.$$

8. Find the volume of the solid bounded below by the xy-plane, above by the surface $z = 1 - x^2 - y^2$ and on the sides by the cylinder $x^2 + y^2 = x$. (Use polar coordinates)