1. (a) If $a_n = \frac{(x-1)^n}{2^n \log(n+1)}$, the ratio $\frac{|a_n|}{|a_{n+1}|} = \frac{|x-2|}{2}\frac{\log(n+1)}{\log(n+2)}$ converges to $\frac{|x-1|}{2}$ as $n \to \infty$. By the ratio test, the series converges for $|x-1| < 2$ and diverges for $|x-1| > 2$. When $x = -1$ the series is $\sum(-1)^n/\log(n+1)$ which converges by the alternating series test. When $x = 3$, the series is $\sum 1/\log(n+1)$ which diverges by comparison with $\sum 1/(n+1)$. Hence $[-1, 3)$ is the interval of convergence.

   (b) Differentiating $\frac{1}{1-x} = \sum_0^\infty x^n$ on $(-1, 1)$, we get $\frac{1}{(1-x)^2} = \sum_1^\infty nx^{n-1} = \sum_0^\infty (n+1)x^n$ so that $g(x) = \frac{4}{(1-x)^2} = \sum_0^\infty 4(n+1)x^n$.

2. (a) Integrating term by term the series expansion $\sin(x^2) = \sum_0^\infty (-1)^{n+1}x^{4n+2}/(2n+1)!$, we get $S = \int_0^1 \sin(x^2)\,dx = \sum_0^\infty (-1)^{n+1}/(4n+3)(2n+1)! = \frac{1}{3} - \frac{1}{7\cdot 3!} + \frac{1}{11\cdot 5!} - \frac{1}{15\cdot 7!} + \cdots$. Since $\frac{1}{11\cdot 5!} < .0005$, we have $S = \frac{1}{3} - \frac{1}{42} + \frac{1}{1320}$ to three decimal places.

   (b) Since $(e^{2x} - 1)^2 = 4x^2 +$ higher degree terms and $\ln(1+x) - x = -x^2/2 +$ higher degree terms, we have
$$\frac{(e^{2x}-1)^2}{\ln(1+x) - x} = \frac{-8 + \text{ higher degree terms}}{1 + \text{ highee degree terms}} \to -8 \text{ as } n \to \infty$$

3. (a) If $F(x, y, z) = \frac{x^2}{y} + \frac{y^2}{z} + \frac{z^2}{x}$, we have $\nabla F(1, 1, 1) = (1, 1, 1)$ which is normal to the surface $F(x, y, z) = 3$ at the point $(1, 1, 1)$. Hence the equation of the tangent plane to this surface at $(1, 1, 1)$ is $x + y + z = 3$.

   (b) The directional derivative of $F(x, y, z)$ at $(1, 1, 1)$ is $\nabla F(1, 1, 1) \cdot (-1, 2, 4)/\sqrt{21} = 5/\sqrt{21}$.

4. (a) Arc length $= s = \int_0^t |\mathbf{r}'(t)|\,dt = \int_0^t \sqrt{5}\,dt = \sqrt{5}t$ so that $\mathbf{r} = (\frac{2s}{\sqrt{5}}, \cos\frac{s}{\sqrt{5}}, \sin\frac{s}{\sqrt{5}})$.

   (b) We have $\mathbf{T} = \frac{d\mathbf{r}}{ds} = \frac{1}{\sqrt{5}}(2, -\sin\frac{s}{\sqrt{5}}, \cos\frac{s}{\sqrt{5}}) = \frac{1}{\sqrt{5}}(2, -\sin t, \cos t)$. Since $\frac{d\mathbf{T}}{ds} = \kappa\mathbf{N}$, where κ is the curvature, we have $\kappa\mathbf{N} = (0, -\sin t, -\cos t)/5$ so that $\kappa = 1/5$ and $\mathbf{N} = (0, -\sin t, -\cos t)$. Hence $\mathbf{B} = \mathbf{T} \times \mathbf{N} = (1, 2\sin t, -2\cos t)/\sqrt{5}$.

5. (a) We have $\frac{\partial f}{\partial x} = 2xy - 2x$, $\frac{\partial f}{\partial y} = x^2 - 2y - 2$ so that $\frac{\partial f}{\partial x} = \frac{\partial f}{\partial y} = 0 \iff (x, y) = (0. -1), (\pm 2, 1)$. These are the critical points of f. Now $A = \frac{\partial^2 f}{\partial x^2} = 2y - 2$, $B = \frac{\partial^2 f}{\partial x \partial y} = 2x$, $C = \frac{\partial^2 f}{\partial y^2} = -2$ so that $D = AC - B^2 = 4 - 4y - 4x^2$. At $(0, -1)$ we have $D = 8 > 0$, $A = -4 < 0$ so that f has a local maximum at $(0, -1)$. At $(\pm 2, 1)$ we have $D = -16 < 0$ so that these two points are saddle points of f.

   (b) The minimum of $d = x^2 + y^2$ occurs at a critical point of $L = x^2 + y^2 - \lambda(xy^2 - 1)$. Since $\frac{\partial L}{\partial x} = 2x - \lambda y^2$ and $\frac{\partial L}{\partial y} = 2y - 2\lambda xy$ we have $\frac{\partial L}{\partial x} = \frac{\partial L}{\partial y} = 0 \iff \lambda = \frac{2x}{y^2} = \frac{1}{x}$, using the fact that $x, y \neq 0$ since $xy^2 = 1$. This gives $2x^2 = y^2$ and hence $2x^3 = 1$ from which $(x, y) = (2^{-1/3}, \pm 2^{1/6})$. The minimum distance is $\sqrt{d} = \sqrt{3}/2^{1/3}$

6. (a) $\int_0^1 \int_{x^{1/3}}^1 \sqrt{1-y^4}dydx = \int_0^1 \int_0^{y^3} \sqrt{1-y^4}dxdy = \int_0^1 y^3\sqrt{1-y^4}dy = 1/6$

   (b) $\iint_{x^2+y^2\le 1} \ln(x^2+y^2)dxdy = \lim_{\epsilon\to 0} \iint_{\epsilon^2\le x^2+y^2\le 1} \ln(x^2+y^2)dxdy = \lim_{\epsilon\to 0}\int_0^{2\pi}\int_\epsilon^1 \ln(r^2)r\,drd\theta = \lim_{\epsilon\to 0} \pi(r^2\ln r^2 - r^2)|_\epsilon^1 = -\pi.$

7. Using cylindrical coordinates, volume $= \int_0^\pi \int_0^{2\sin\theta} \int_0^{r^2} r\,dzdrd\theta = \int_0^\pi \int_0^{2\sin\theta} r^3\,drd\theta = 4\int_0^\pi \sin^4\theta d\theta = 3\pi/2.$

8. Projecting onto the xz-plane, $I = \iiint_R xz\,dV = \int_0^1 \int_0^{1-x} \int_z^{x+z} xz\,dydzdx = \int_0^1 \int_0^{1-x} x^2z\,dzdx = 1/60.$ Projecting onto the yz-plane, $I = \int_0^1 \int_0^y \int_{y-z}^{1-z} xz\,dxdzdy.$