1. (a) Find the interval of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{(x-1)^n}{2^n \log(n+1)}.$$

(b) Find a power series representation about the point $x = 0$ for

$$g(x) = \frac{4}{(1-x)^2}.$$

2. (a) Using a power series expansion for the sine function, compute

$$\int_0^1 \sin(x^2)\, dx$$

to 3 decimal places.

(b) Compute

$$\lim_{x \to 0} \frac{(e^{2x}-1)^2}{\ln(1+x) - x}$$

3. (a) Find the equation of the tangent plane to the surface

$$\frac{x^2}{y} + \frac{y^2}{z} + \frac{z^2}{x} = 3$$

at the point $(1, 1, 1)$.

(b) Find the directional derivative of the function

$$F(x, y, z) = \frac{x^2}{y} + \frac{y^2}{z} + \frac{z^2}{x}$$

at the point $(1, 1, 1)$ in the direction $(-1, 2, 4)$.

4. (a) Reparametrize the curve

$$\mathbf{r}(t) = (2t, \cos t, \sin t)$$

in terms of arc length measured from the point where $t = 0$.

(b) For the curve in (a), find the unit tangent, unit principal normal and binormal vectors **T,N,B** of the Frenet-Serret formulas as well as the curvature at any point on the curve.

5. (a) Find and classify the critical points of

$$f(x,y) = x^2y - x^2 - y^2 - 2y$$

as local maxima, local minima or saddle points using the test involving the second partial derivatives of $f(x,y)$.

(b) Use the Lagrange multiplier method to find the shortest distance from the origin to the curve $xy^2 = 1$.

6. For each of the following double integrals

$$\text{(a)} \quad \int_0^1 \int_{x^{1/3}}^1 \sqrt{1-y^4}\, dy\, dx, \qquad \text{(b)} \iint_{x^2+y^2\leq 1} \ln(x^2+y^2)\, dxdy,$$

sketch the domain of integration and evaluate the integral.

7. Find the volume of the region bounded by the cylinder $x^2 + y^2 = 2y$, the paraboloid $x^2 + y^2 = z$ and the plane $z = 0$.

8. Compute $\iiint_R xz\, dV$, where R is the solid tetrahedron with vertices

$$(0,0,0), (1,0,0), (1,1,0), (0,1,1).$$