1. The point \((x, y)\) is a critical point of the function \(f(x, y)\) if and only if \(\frac{\partial f}{\partial x} = \frac{\partial f}{\partial y} = 0\). Now \(\frac{\partial f}{\partial x} = 3e^y - 3x^2\) and \(\frac{\partial f}{\partial y} = 3xe^y - 3e^y\) so that \((x, y)\) is a critical point if and only if \(e^y = x^2\) and \(x = e^{2y}\). These two equations have the unique solution \(x = 1, y = 0\). Now \(A = \frac{\partial^2 f}{\partial x^2} = -6x, B = \frac{\partial^2 f}{\partial x \partial y} = 3e^y, C = 3e^y - 9e^{3y}\) so that at the critical point \((1, 0)\) we have \(A < 0, AC - B^2 = (-6)\cdot(-6) - 9 = 27 > 0\) which shows that \(f(1, 0) = 1\) is a local maximum. Since \(f(-3, 0) = 17\) the function \(f\) does not have a maximum at \((1, 0)\).

2. Since the function \(f(x, y) = 2x + 3y\) is continuous it has a maximum on the ellipse \(x^2 + xy + 2y^2 = 37\). This maximum is a critical point of \(L = 2x+3y-\lambda(x^2+xy+2y^2-37)\). Since \(\frac{\partial L}{\partial x} = 2 - \lambda(2x+y)\) and \(\frac{\partial L}{\partial y} = 3 - \lambda(x+4y)\), the critical points of \(l\) satisfy \(\lambda(2x+y) = 2, \lambda(x+4y) = 3, x^2 + xy + 2y^2 = 37\). Eliminating \(\lambda\) in the first two equations gives \(y = 4x/5\). Substituting this in the third equation gives \(x = 5\sqrt{37/77}, y = 4\sqrt{37/77}\) and \(x = -5\sqrt{37/77}, y = -4\sqrt{37/77}\). The function \(f\) has the maximum \(f(5\sqrt{37/77}, 4\sqrt{37/77}) = 22\sqrt{37/77}\) on the given curve. It takes its minimum value \(-22\sqrt{37/77}\) at \((-5\sqrt{37/77}, -4\sqrt{37/77})\).

3. Since the first integral is improper at the lower limit, we have

\[
\int_0^2 \int_0^{\ln y} \frac{\ln y}{\sqrt{y}} dy \, dx = \lim_{\epsilon \to 0} \int_0^2 \int_{\epsilon}^{\ln y} \frac{\ln y}{\sqrt{y}} dy \, dx = \lim_{\epsilon \to 0} \int_{\epsilon}^{\ln y} \frac{\ln y}{\sqrt{y}} \, dy \, dx
\]

\[
= \lim_{\epsilon \to 0} \left[ \frac{2}{3} y^{3/2} \ln y - \frac{2}{3} \int_{\epsilon}^{\ln y} \sqrt{y} \, dy - 2\epsilon \sqrt{y} \ln y \right] + 2\epsilon \left[ \int_{\epsilon}^{2} \frac{dy}{\sqrt{y}} \right]
\]

\[
= \lim_{\epsilon \to 0} \left[ \frac{2}{3} y^{3/2} \ln y - \frac{4}{9} y^{3/2} - 2\epsilon \sqrt{y} \ln + 4\epsilon \sqrt{y} \right] \bigg|_{\epsilon}
\]

\[
= \frac{4}{9} \sqrt{2} (3 \ln 2 - 2), \text{ using l'Hospital's Rule (lim y^a ln y = 0 for } \alpha > 0).\]

4. Volume = \(\int_0^{2\pi} \int_1^2 (r \cos \theta + 2) \, r \, dr \, d\theta = 6\pi\).