1. Given the curve $x = t^3/3$, $y = 2t$, $z = 2/t$, $(t > 0)$, find
   
   (a) the unit tangent, principal normal and curvature at any point;

   (b) the equation of the tangent line to the curve at the point where $t = 1$.

2. If $u = f(r)$, where $f$ is differentiable and $r = \sqrt{x^2 + y^2 + z^2}$, show that, for $(x, y, z) \neq (0, 0, 0)$,

   (a) $\left( \frac{du}{dr} \right)^2 = \left( \frac{\partial u}{\partial x} \right)^2 + \left( \frac{\partial u}{\partial y} \right)^2 + \left( \frac{\partial u}{\partial z} \right)^2$

   and

   (b) $\nabla u = \frac{1}{r \, dr} (x\vec{i} + y\vec{j} + z\vec{k})$.

3. (a) Find the equation of the tangent plane and normal line to the surface $z = 3xe^y - x^3 - e^{3y}$ at the point $(0, 0, -1)$.

   (b) Find the equation of the tangent plane to the surface

   $x = 2s^2 + t^3$, $y = s^2t^3$, $z = s^2 - st^3$

   at the point where $s = t = 1$.

4. Suppose that $T(x, y, z) = x^3y + y^3z + z^3x$ is the temperature at the point $(x, y, z)$ in 3-space.

   (a) Calculate the directional derivative of $T$ at the point $P(2, -1, 0)$ in the direction from $P$ to the point $Q(1, 1, 2)$.

   (b) A mosquito is flying through space with constant speed 5 in the direction of increasing temperature. If the mosquito’s direction of flight at any given point is always normal (perpendicular) to the level surface of $f(x, y, z) = 2x^2 + 3y^2 + z^2$ passing through this point, find the rate of change of temperature experienced by the mosquito when it is at the point $(2, -1, 0)$. 

JUSTIFY ALL YOUR ASSERTIONS