1. (a) Determine whether the following series converge or diverge.

(i) \[ \sum_{n=0}^{\infty} \frac{\cos(n\pi)}{\sqrt{n+1}} \],

(ii) \[ \sum_{n=1}^{\infty} \frac{n(n+6)}{(n+1)(n+3)(n+5)} \].

(b) Find the values of \( p \) for which the series

\[ \sum_{n=1}^{\infty} \frac{\ln n}{n^p} \]

is convergent.

2. (a) Let

\[ f(x) = \sum_{n=1}^{\infty} \frac{(x-3)^n}{5^n \sqrt{n^3}} \],

defined on the interval of convergence \( I \) of the series. Find \( I \) and the radius of convergence \( R \) of the series.

(b) Find a power series representation of \( f'(x) \). For which values of \( x \) is this representation valid?

3. (a) Expand

\[ f(x) = \frac{x + x^2}{(1-x)^3} \]

as a power series. [Hint: Write \( f(x) \) as \( \frac{x}{(1-x)^3} + \frac{x^2}{(1-x)^3} \).]

(b) Use part (a) to find the sum of the series

\[ \sum_{n=1}^{\infty} \frac{n^2}{2^n} \].

4. Given the curve \( \vec{r}(t) = (t + 1, 2\sqrt{2}t^{\frac{3}{2}} + 362, \frac{1}{4}t^2) \), \( t > 0 \), find

(a) the equation of the tangent line to \( C \) at \( t = 1 \);

(b) the arc length of \( C \) between \( t = 1 \) and \( t = 2 \).

Estimated time for completion of this assignment is 90 minutes.