McGill University
Math 222A: Calculus III
Assignment 1: due in class Friday, September 25, 1998

This assignment covers sections $11.2, 11.3, 11.5, 11.6, 11.7$ **of the text.**

1. Determine whether the sequence $(a_n)_{n\geq 1}$ converges and find its limit if it does converge.

$$(a)\ \ a_n = \frac{2^n - 3^n}{5^n}, \quad (b)\ \ a_n = (1 - \frac{3}{n})^{n+1}, \quad (c)\ \ a_n = \sqrt{n}\sin(\frac{1}{n}), \quad (d)\ \ a_n = (-1)^n \frac{n^2+1}{n^2-1}.$$

2. Use a comparison test to determine whether the given series converges or diverges.

$$(a)\ \ \sum_{n=1}^{\infty}(\frac{n-3}{n})^{n+1}, \quad (b)\ \ \sum_{n=1}^{\infty}\frac{5n^2+5}{3n^4-1}, \quad (c)\ \ \sum_{n=1}^{\infty} n^{-(1+1/n)}, \quad (d)\ \ \sum_{n=1}^{\infty} n\ \sin(\frac{1}{n^3}).$$

3. Use the ratio or root test to determine whether the given series converges or diverges.

$$(a) \sum_{n=1}^{\infty}(1 - \frac{1}{n})^{n^2}, \qquad (b)\ \ \sum_{n=0}^{\infty}\frac{n!\,(n+1)^3}{(2n)!}.$$

4. Determine whether the given series converges or diverges and, in the case of convergence, whether the converges is absolute or conditional.

$$(a)\ \ \sum_{n=1}^{\infty}(-1)^n(1-\frac{1}{n})^n, \quad (b)\ \ \sum_{n=1}^{\infty}\cos(n)\sin(\frac{1}{n^2}), \quad (c)\ \ \sum_{n=1}^{\infty}(-1)^n\frac{\ln n}{\sqrt{n}}, \quad (d)\ \ \sum_{n=2}^{\infty}(-1)^n\frac{1}{n\,(\ln n)^{3/2}}.$$

5. Find the sum of the given series, correct to 3 decimal places after rounding.

$$(a)\ \ \sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n\,10^n}, \qquad (b)\ \ \sum_{n=1}^{\infty}\frac{1}{n^5-1}.$$