MATH 133: Vectors, Matrices and Geometry

Solution to Written Assignment 2

Solution. Applying Gauss-Jordan reduction to the augmented matrix of the system, we get

$$\begin{bmatrix} t & 1 & 1 & 1 \\ 1 & t & 1 & 1 \\ 1 & 1 & t & 1 \end{bmatrix} R_1 \leftrightarrow R_3 \begin{bmatrix} 1 & 1 & t & 1 \\ 1 & t & 1 & 1 \\ t & 1 & 1 & 1 \end{bmatrix} \begin{matrix} R_2 - R_1 \\ R_3 - tR_1 \end{matrix} \begin{bmatrix} 1 & 1 & t & 1 \\ 0 & t-1 & 1-t & 0 \\ 0 & 1-t & 1-t^2 & 1-t \end{bmatrix} R_3 + R_2 \begin{bmatrix} 1 & 1 & t & 1 \\ 0 & t-1 & 1-t & 0 \\ 0 & 0 & 2-t-t^2 & 1-t \end{bmatrix}$$

Case 1: $t \neq 1$. Continuing the reduction process when $t \neq 1$, we get

$$\begin{matrix} \frac{1}{t-1} R_2 \\ \frac{1}{1-t} R_3 \end{matrix} \begin{bmatrix} 1 & 1 & t & 1 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & t+2 & 1 \end{bmatrix}$$

If $t = -2$, the system is inconsistent since we get the equation $0 = 1$. If $t \neq -2$, the system has the unique solution $x = y = z = \frac{1}{t+2}$. In this case, the system represents three planes which intersect in a single point.

Case 2: t=1. In this case our system consists of the single equation $x + y + z = 1$, which is a plane. It has the solutions

$$x = 1 - s - t, \quad y = s, \quad z = t,$$

with s, t arbitrary scalars.