MATH 133: Vectors, Matrices and Geometry

Written Assignment 6

Due in tutorial or by Friday 1pm the week of November 10, 2003

Write the name of your tutor and your tutorial section number

in the top right corner of the first page.

Justify all of your assertions.

Problem

(a) The trace of an $n \times n$ matrix $A = [a_{ij}]$ is defined to be $\operatorname{tr}(A) = a_{11} + a_{22} + \cdots a_{nn}$, the sum of the diagonal elements of A. Show that $\operatorname{tr}(a\,A + b\,B) = a\operatorname{tr}(A) + b\operatorname{tr}(B)$ and that $\operatorname{tr}(AB) = \operatorname{tr}(BA)$. Use this to show that there are no $n \times n$ matrices A, B with $AB - BA = I$.

(b) If A is a 2×2 matrix, show that $A^2 - \operatorname{tr}(A)A + \det(A)I = 0$.

(c) Show that the n-dimensional column vector

$$\begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}$$

is an eigenvector of the $n \times n$ matrix A if and only if the rows sums of A are constant.

(d) The numbers 77843, 53227, 25755, 20927 and 78421 are divisible by 17. Show, without computing the determinant, that

$$\begin{vmatrix} 7 & 7 & 8 & 4 & 3 \\ 5 & 3 & 2 & 2 & 7 \\ 2 & 5 & 7 & 5 & 5 \\ 2 & 0 & 9 & 2 & 7 \\ 7 & 8 & 4 & 2 & 1 \end{vmatrix}$$

is also divisible by 17. **Hint:** Use the decimal representation of the given 5 numbers.