MATH 133: Vectors, Matrices and Geometry

Written Assignment 5

Due in tutorial or by Friday 1pm the week of October 27, 2003

Write the name of your tutor and your tutorial section number

in the top right corner of the first page.

Justify all of your assertions.

Problem. Let A and B be $m \times p$ and $p \times n$ matrices respectively.

(a) Show that the column space of AB is a subset of the column space of A. Show how to deduce that $\text{rank}(AB) \leq \text{rank}(A)$;

(b) Show that $\text{rank}(AB) \leq \min(\text{rank}(A), \text{rank}(B))$; (Hint: Use transposes.)

(c) If C is a matrix with $BC = I$, the identity $p \times p$ matrix, show that $\text{rank}(AB) = \text{rank}(A)$; Hint: Use the fact that $BC = I \implies B(CY) = Y$ for any Y in $\mathbb{R}^p$.)

(d) Give an example with $m = n = p = 2$ where $\text{rank}(AB) = \text{rank}(A) \neq 0$ and $B^2 = 0$.

Bonus Problem.

(a) If $B = \begin{bmatrix} 1 & 2 & -1 & 3 \\ 1 & 3 & 0 & 1 \end{bmatrix}$, find a matrix C with $BC = I$.

(b) If B is a $p \times n$ matrix, show that there is a matrix C with $BC = I \iff \text{rank}(B) = p$. (Hint: Show that if $\text{rank}(B) = p$, the system of equations $BX = Y$ is solvable for any Y in $\mathbb{R}^p$.)