Written Assignment 5

Due in tutorial or by Friday 1pm the week of October 27, 2003
Write the name of your tutor and your tutorial section number
in the top right corner of the first page.
Justify all of your assertions.

Problem. Let A and B be $m \times p$ and $p \times n$ matrices respectively.

(a) Show that the column space of AB is a subset of the column space of A. Show how to deduce that rank$(AB) \leq$ rank(A);

(b) Show that rank$(AB) \leq$ min(rank(A), rank(B)); (Hint: Use transposes.)

(c) If C is a matrix with $BC = I$, the identity $p \times p$ matrix, show that rank$(AB) =$ rank(A); Hint: Use the fact that $BC = I \implies B(CY) = Y$ for any Y in $\mathbb{R}^p$.)

(d) Give an example with $m = n = p = 2$ where rank$(AB) =$ rank$(A) \neq 0$ and $B^2 = 0$.

Bonus Problem.

(a) If $B = \begin{bmatrix} 1 & 2 & -1 & 3 \\ 1 & 3 & 0 & 1 \end{bmatrix}$, find a matrix C with $BC = I$.

(b) If B is a $p \times n$ matrix, show that there is a matrix C with $BC = I \iff$ rank$(B) = p$. (Hint: Show that if rank$(B) = p$, the system of equations $BX = Y$ is solvable for any Y in $\mathbb{R}^p$.)