Department of Mathematics and Statistics

MATH 133: Vectors, Matrices and Geometry

Test 2

Due by 3 pm Friday November 21, 2003.

Justify all of your assertions.

Make sure the course number, your name, student number and the name of your TUTORIAL INSTRUCTOR are at the top of each page.

Let $A = \begin{bmatrix} 5 & 3 & 1 \\ 3 & 3 & 3 \\ 1 & 3 & 5 \end{bmatrix}$.

1. Find the characteristic polynomial of A and the eigenvalues of A.
2. Find a basis for each eigenspace of A.
3. Find an invertible matrix P such that $P^{-1}AP$ is a diagonal matrix.
4. Find a matrix B with $B^2 = A$. Show that there are at least 4 such matrices.
5. If B is a matrix with $AB = BA$, show that the eigenvectors of A are also eigenvectors of B. (Hint: If $AX = cX$, show that $A(BX) = cBX$ and use the fact that the eigenspaces of A are 1-dimensional.) Use this to deduce that there are exactly 4 matrices with $B^2 = A$ and $AB = BA$.