Department of Mathematics and Statistics

MATH 133: Vectors, Matrices and Geometry

Test 1

Due by 1 pm Friday October 24, 2003

Justify all of your assertions

Make sure the course number, your name, student number and instructor's name are at the top of each page

1. Let $P(1,-2,3), A(2,1,-1), B(3,-1,4)$ be given points in $\mathbb{R}^3$ and let L be the line passing through A and B.

   (a) Find parametric equations for the line L;

   (b) Find the distance of the point P to the line L;

   (c) Find the coordinates of the point Q on L which is closest to P;

   (d) Find the equation of the plane perpendicular to L and passing through P.

2. Let $x-2y+3z=2,\ 2x+y-3z=3$ be given planes in $\mathbb{R}^3$ and let L be the line of intersection of these two planes.

   (a) Find parametric equations for L;

   (b) Find the equation of the plane containing L and the point $P(1,-1,1)$;

   (c) If $ax+by+cz=d$ is the equation of the plane found in 2(b), show that the vector (a,b,c,d) is a linear combination of the vectors $(1,-2,3,2),\ (2,1,-3,3)$.

3. Consider the system of equations

$$\begin{aligned} x_1 + cx_2 + 2x_3 + cx_4 &= 1 \\ 2x_1 + 2cx_2 + 5x_3 + x_4 &= c+1 \\ cx_1 + c^2x_2 + 3cx_3 - 2x_4 &= c+2 \end{aligned}$$

   where c is a parameter.

   (a) Find the reduced echelon form of the augmented matrix of the given system;

   (b) What are the values of c for which the given system of equations is consistent;

   (c) Find a parametric description of the solution set of the given system for each value of c found in 3(b).

   Note that your answers to 3(a) and 3(c) depend on the value of c.