We now derive the parametric equations of a line L in space from its vector equation.

$\vec{d}$ = direction of L

$= [\alpha, \beta, \gamma]$

$\alpha = d - a,\ \beta = e - b,\ \gamma = f - c$

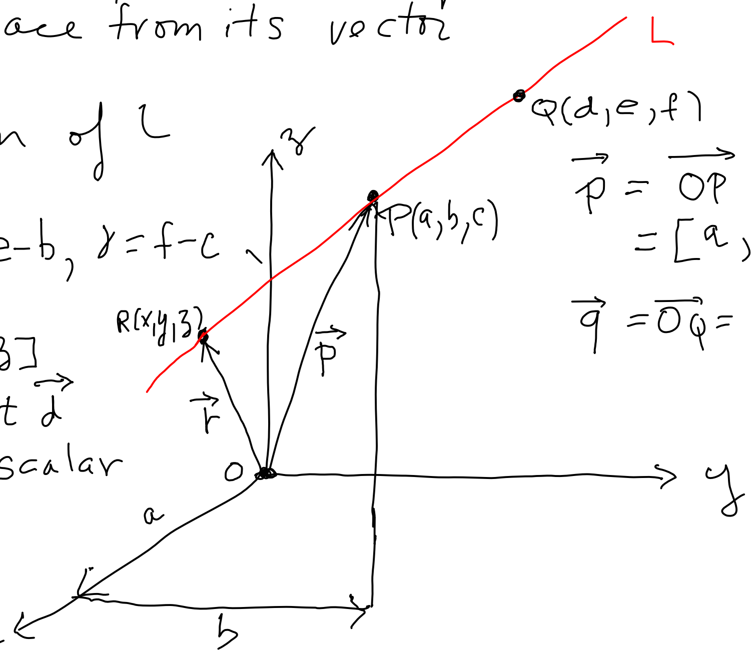
$\vec{r} = \overrightarrow{OR} = [x, y, z]$

$\overrightarrow{PR} = t\,\overrightarrow{PQ} = t\,\vec{d}$

t some scalar

$\vec{r} = \vec{p} + t\vec{d}$

vector equation of L



$\vec{p} = \overrightarrow{OP}$

$= [a, b, c]$

$\vec{q} = \overrightarrow{OQ} = [d, e, f]$

$\vec{r} = [x, y, z]$, $\vec{p} = [a, b, c]$, $\vec{d} = [\alpha, \beta, \gamma]$

$$[x, y, z] = [a, b, c] + t[\alpha, \beta, \gamma]$$
$$= [a, b, c] + [t\alpha, t\beta, t\gamma]$$
$$= [a + t\alpha, b + t\beta, c + t\gamma]$$

i.e. $x = a + t\alpha$, $y = b + t\beta$, $z = c + t\gamma$

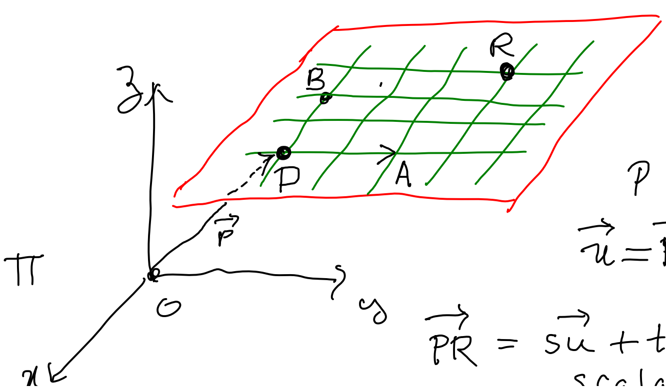
t = arbitrary scalar

These are parametric equations for L

We now derive the vector equation of a plane Π

P, A, B are given points of the plane Π

$R(x, y, z)$ any point of Π



$\vec{p} = \overrightarrow{OP}$

$= [a, b, c]$

P has coord (a, b, c)

$\vec{u} = \overrightarrow{PA}$, $\vec{v} = \overrightarrow{PB}$

$\overrightarrow{PR} = s\vec{u} + t\vec{v}$ for some scalars s, t

$R(x,y,z)$ any point of Π

$\overrightarrow{PR} = s\vec{u} + t\vec{v}$ for some scalars s, t

$$\overrightarrow{OR} = \overrightarrow{OP} + s\vec{u} + t\vec{v}$$

$$\vec{u} = [\alpha_1, \beta_1, \gamma_1], \quad \vec{v} = [\alpha_2, \beta_2, \gamma_2]$$

$$\overrightarrow{OR} = [x, y, z]$$

$$[x, y, z] = [a, b, c] + s[\alpha_1, \beta_1, \gamma_1] + t[\alpha_2, \beta_2, \gamma_2]$$

$$x = a + s\alpha_1 + t\alpha_2$$
$$y = b + s\beta_1 + t\beta_2$$
$$z = c + s\gamma_1 + t\gamma_2$$

The are the parametric equations of a plane

Example. Find the parametric equations of a plane passing through
$P(1,2,1)$, $A(3,4,2)$, $B(2,1,3)$

$$\vec{u} = \overrightarrow{PA} = [2, 2, 1] \qquad \vec{P} = [1, 2, 1]$$
$$\vec{v} = \overrightarrow{PB} = [1, -1, 2]$$

$$[x,y,z] = [1, 2, 1] + s[2, 2, 1] + t[1, -1, 2]$$

$$[x, y, z] = [1 + 2s + t, 2 + 2s - t, 1 + s + 2t]$$

$$x = 1 + 2s + t$$
$$y = 2 + 2s - t$$
$$z = 1 + s + 2t$$

Algebraic Properties of vectors

$\vec{u}, \vec{v}, \vec{w}$ $\quad a, b, c$ scalars

V1 Associative Law: $(\vec{u} + \vec{v}) + \vec{w} = \vec{u} + (\vec{v} + \vec{w})$

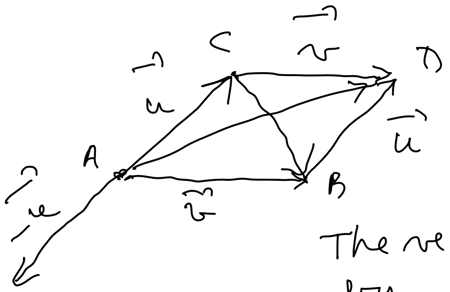
V2. Commutative Law: $\vec{u} + \vec{v} = \vec{v} + \vec{u}$

V3. Existence of a zero vector: There is a vector $\vec{0}$ such that $\vec{u} + \vec{0} = \vec{u}$

V4. Existence of additive inverses: Given $\vec{u}$ there is a vector $\vec{v}$ such that $\vec{u} + \vec{v} = \vec{0}$

$\vec{0}$ such that $\vec{u}+\vec{0}=\vec{u}$

V4. Existence of additive inverses: Given $\vec{u}$ there is a vector $\vec{v}$ such that $\vec{u}+\vec{v}=0$. This vector is denoted by $-\vec{u}$.



$\vec{AD} = \vec{u}+\vec{v} = \vec{v}+\vec{u}$

If we let $\vec{w}=\vec{CB}$ then $\vec{u}+\vec{w}=\vec{v}$ by Δ law

The vector $\vec{w}$ is also denoted by $\vec{v}-\vec{u}$. This is the definition of subtraction of vectors.

If $\vec{u}=[a,b]$, $\vec{v}=[c,d]$ then

$\vec{u}-\vec{v} = [c-a, d-b]$ since

$[a,b]+[c-a,d-b] = [$

V5. $1\vec{u} = \vec{u}$

V6. $(ab)\vec{u} = a(b\vec{u})$

V7. $(a+b)\vec{u} = a\vec{u}+b\vec{u}$ } Distributive Laws

$a(\vec{u}+\vec{v}) = a\vec{u}+a\vec{v}$

Exercise. Show that the medians of a triangle meet in a point P which divides each median in the ratio 2 : 1. This point is called the centroid of the triangle.