1. (12 marks) Consider the three points $P = (4, 0, 1)$, $Q = (2, 1, 1)$ and $R = (-1, -2, -3)$ in $\mathcal{R}^3$.

   (a) Find an equation of the plane containing the points P, Q and R.

   (b) Find the area of a parallelogram having P, Q and R as three of its four vertices.

   (c) Find an equation of the line through Q and R.

   (d) Find the distance from P to the line through Q and R.

2. (8 marks) If A and B are 4×4 matrices such that $\det(A) = -2$ and $\det(B) = 1$, then what is $\det(A^3(2B)^{-1}(A^T)^2(-A)^{-1})$?

ANSWER:

3. (8 marks) This problem concerns linear transformations of the plane $\mathcal{R}^2$.

   (a) If T_1 is the counterclockwise rotation through an angle of $\frac{\pi}{2}$, what is the standard matrix of T?

   (b) If T_2 is the reflection across the line $y = -x$, what is the standard matrix of T_2?

   (c) If $T_3 = T_2 \circ T_1$, what is the standard matrix of T_3?

   (d) Show that T_3 is the reflection across a line, and find that line.

4. (10 marks) Find an invertible matrix P and a diagonal matrix D such that $P^{-1}AP = D$, where $A = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \end{pmatrix}$.

$P =$

$D =$

5. (5 marks) Suppose that P is invertible and $P^{-1}AP = B$, where A, B and P are square matrices of the same size. Show that A and B have the same characteristic polynomial.

6. (10 marks) Suppose that $\{\vec{v}_1, \vec{v}_2\}$ is an independent set of vectors and that $\vec{w}_1 = a\vec{v}_1 + b\vec{v}_2$, $\vec{w}_2 = c\vec{v}_1 + d\vec{v}_2$ for some scalars a, b, c and d. Show that $\{\vec{w}_1, \vec{w}_2\}$ is independent if and only if the matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is invertible.

7. (10 marks) Find all constants k such that the set $\left\{ \begin{pmatrix} k \\ k \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ k \end{pmatrix}, \begin{pmatrix} -k \\ 0 \\ 1 \end{pmatrix} \right\}$ is a basis for $\mathcal{R}^3$.

8. (12 marks) Let $A = \begin{pmatrix} 1 & 1 & 1 & 0 & 2 \\ 2 & 2 & 2 & -3 & 5 \\ 3 & 3 & 3 & -3 & 7 \end{pmatrix}$. Find a basis for the row space, column space, and null space of A.

(a) Row space:

(b) Column space:

(c) Null space:

9. (8 marks) Express $\begin{pmatrix} 1 & 1 \\ 2 & -2 \end{pmatrix}$ as a product of elementary matrices.

10. (10 marks) Find an orthonormal basis for $\mathcal{R}^3$ containing the vector $\begin{pmatrix} \frac{1}{3} \\ \frac{2}{3} \\ \frac{2}{3} \end{pmatrix}$.

11. (7 marks) Is it possible that a 2 x 2 matrix with trace 6 and determinant 9 is diagonalizable but not diagonal? (Circle YES or NO and justify your answer.)

YES, because

NO, because

(Please continue rough work here)

(Please continue rough work here)

(Please continue rough work here)

(Please continue rough work here)

NAME: STUDENT NUMBER:

McGILL UNIVERSITY

FACULTY OF SCIENCE

FINAL EXAMINATION

MATH 133

Vectors Matrices and Geometry

Examiner: Professor J. Loveys Date: Tuesday, April 15, 2003
Associate Examiner: R. Mohammadalikhani Time: 9:00 A.M. - 12:00 P.M.

INSTRUCTIONS

Answer all questions on the pages provided.
If your final answer is not in the obvious place, please indicate clearly WHAT it is, and WHERE it is.
Indicate clearly any rough work you wish considered for marks, and which question this rough work is for.
The last four pages are for rough work.
The value of each question is indicated next to the question; the examination is worth a total of 100 marks.
Show all your work.
Calculators are not permitted.
Dictionaries are not permitted.
This is a closed book exam.

This exam comprises the cover and 11 pages with 11 questions and 4 pages for Rough work.