Theorem (Invertible Matrix Theorem, Full Version). *Let A be an $n \times n$ matrix. Then the following statements are equivalent.*

(a) *A is invertible.*

(b) *The homogeneous system $AX = 0$ has only the trivial (zero) solution.*

(c) *A can be carried to the identity matrix I by elementary row operations.*

(d) *The system $AX = B$ has a solution for every $n \times 1$ column vector B.*

(e) *There is an $n \times n$ matrix C such that $AC = I$ (i.e. A has a right inverse).*

(f) *There is an $n \times n$ matrix C such that $CA = I$ (i.e. A has a left inverse).*

(g) *A^T is invertible.*

(h) $\text{rank}(A) = n$.

(i) *A can be expressed as a product of elementary matrices.*

(j) $\det(A) \neq 0$.

(k) *0 is not an eigenvalue of A.*

(l) *The columns of A are linearly independent in $\mathbb{R}^n$.*

(m) *The columns of A span $\mathbb{R}^n$.*

(n) *The rows of A are linearly independent in $\mathbb{R}^n$.*

(o) *The rows of A span $\mathbb{R}^n$.*