MATH 578 - PROBLEM SET 1

Posted	Tuesday, October 4^{th} 2010
Due	Tuesday, October 18^{th} 2010

Problem 1

The purpose of this exercise is to verify numerically the order of accuracy of the Lagrange interpolation technique.

Recall the following:

Theorem. *Suppose $f \in C^n([a,b])$. Let $p(x)$ be the unique polynomial inP_{n-1} interpolating $f(x)$ at the nodes $\{x_j\}$ with $a = x_1 < x_2 < \cdots < x_n = b$. Then, for each $x \in [a,b]$ there is a $\xi \in [a,b]$, such that,*

$$f(x) - p(x) = \frac{f^{(n)}(\xi)}{n!}(x - x_1)\cdots(x - x_n)$$

and if $h = \max_j (x_{j+1} - x_j)$,

$$\|f - p\|_\infty = \sup_{a \le x \le b} |f(x) - p(x)| \le \frac{h^n}{4n} \|f^n\|_\infty .$$

Part A (50%). Write a Matlab code that will allow you to estimate the error $E = \|f - p\|_\infty$ (using a constant h) for various degrees $n = \{2, 7, 16\}$ of the interpolation polynomial $p(x)$ and that for each function $f(x) = \sin(x)$, and $f(x) = x^6$.

for each case, plot E versus h in loglog, and produce a table in which you will present the numerically estimated order of accuracy of the method.

Discuss your results briefly, and don't forget to print your code (please be neat!).

Part B (Bonus 25%). Repeat Part A *but* using Chebyshev polynomials instead of Lagrange polynomials.

Problem 2

The Fourier transform (12.5% **each).**

- Prove that if $u, v \in L^2$,
$$(u * v)(x) = (v * u)(x).$$
- Compute $\widehat{u_{(p)}}(k) \equiv F(u * u * \cdots *)(k)$ for the function
$$u(x) = \begin{cases} \frac{1}{4} & -2 \le x < 0 \\ -\frac{1}{4} & 0 < x \le 2 \\ 0 & \textit{otherwise.} \end{cases}$$
- Compute the Fourier transform of the Gaussian function
$$u(x; s) \equiv \frac{1}{\sqrt{2\pi s^2}} \exp\left(-\frac{x^2}{2s^2}\right).$$
- Prove that the convolution of p Gaussian functions (with variances $s_1^2, s_2^2, \cdots, s_p^2$) is a Gaussian function with $s = \sqrt{\sum s_i^2}$.

Problem 3

Write a one-page project proposal.