PROBLEM SET 2

The problems are taken from Chapter 3 of the Lecture Notes by W. W. L. Chen, located at

`http://www.maths.mq.edu.au/~wchen/lnentfolder/ent03-c.pdf`

Problem 1. Chen, chapter 3, # 2. Prove that every year, including a leap year, has Friday the 13th (and also Sunday the 1st).
Problem 2. Chen, chapter 3, # 4. Show that $1^2, 2^2, \ldots, m^2$ is not a complete set of residues modulo m if $m > 2$.
Problem 3. Chen, chapter 3, # 5. Suppose that $a, b, p \in \mathbf{N}$ and p is prime. Show that $(a+b)^p \equiv a^p + b^p (\mathrm{mod} p)$. Hint: use the *binomial formula*:

$$(a+b)^p = a^p + \sum_{k=1} p-1 \binom{p}{k} a^k b^{p-k} + b^p.$$

Problem 4. Chen, chapter 3, # 7.
Suppose that $p > 2$ is a prime. Show that $1^p + 2^p + \ldots + (p-1)^p \equiv 0 (\mathrm{mod} p)$.