PROBLEM SET 1

The problems are taken from Chapter 1 of the Lecture Notes by W. W. L. Chen, located at

`http://www.maths.mq.edu.au/~wchen/lnentfolder/ent01-df.pdf`

Problem 1. Chen, chapter 1, # 2. Prove that if $n \in \mathbf{N}$ is composite, then n has a prime factor not exceeding $\sqrt{n}$.

Problem 2. Chen, chapter 1, # 4. Prove that the three natural numbers $n, n+2, n+4$ cannot be simultaneously prime unless $n = 3$.

Problem 3. Chen, chapter 1, # 6. Prove that $24|n(n^2-1)$ for every odd $n \in \mathbf{N}$.

Problem 4. Chen, chapter 1, # 8. Suppose that $a, b, c \in \mathbf{N}$.

(i) Prove that if $3|(a^2+b^2)$, then $3|ab$.

(ii) Prove that if $9|(a^3+b^3+c^3)$, then $3|abc$.

Problem 5. Chen, chapter 1, # 11(i). Suppose that p is a prime.

Prove that $\binom{p}{k}$ is divisible by p for every $k = 1, 2, ..., p-1$. Here

$$\binom{p}{k} = \frac{p!}{k!(p-k)!} = \frac{p(p-1)\cdot \ldots \cdot (p-k+1)}{1 \cdot 2 \cdot \ldots \cdot k}$$

is a *binomial coefficient*; we shall discuss them in a future lecture.