Math 581, Winter 2018 D. Jakobson

EXTRA CREDIT PROBLEMS due date to be announced

Every problem is worth 5 points.

Problem 1. Let $F(x)$ be the *Cantor staircase* function, i.e. F is monotone and continuous on $[0,1]$, constant on each interval in the complement of K, and takes values $\{1/2^m, 3/2^m, 5/2^m \ldots, (2^m-1)/2^m\}$ on the m-th generation intervals. Compute the Lebesgue-Stieltjes integrals

a) $\int_0^1 x^k dF(x)$;

b) $\int_0^1 e^x dF(x)$;

c) $\int_0^1 \sin(\pi x) dF(x)$.

Hint: relate $F(x/3)$ to $F(x)$, and also $F((x+2)/3)$ to $F(x)$. Show that $\int_{1/3}^{2/3} h(x)dF(x)$ for any function h. Next, for functions $h(x)$ in a), b) and c), find a recursion between $\int_0^1 h(x)dF(x)$, $\int_0^{1/3} h(x)dF(x)$ and $\int_{2/3}^1 h(x)dF(x)$. After that, iterate.