Math 581, Winter 2018 D. Jakobson

PROBLEM SET 2 Due date to be announced

Do all the problems. Every problem is worth 5 points. Some problems will not be graded because of time constraints.

Evans, Chapter 7, No. 2, 4, 6, 7, 10, 12.

SP 3. Use min-max characterization of eigenvalues to prove *Courant nodal domain* theorem: let ϕ_k be the k-th eigenfunction of the Dirichlet boundary value problem in an open, bounded, connected domain $U \subset \mathbb{R}^n$. Then ϕ_k has $\leq k$ *nodal domains:* connected components of the complement of the nodal set of ϕ_k, i.e. of the set $\{x \in U : \phi_k(x) \neq 0\}$. Hint: Assume for contradiction that ϕ_k has at least $(k+1)$ nodal domains $A_1, \ldots, A_{k+1}, \ldots$. Use $\phi_k \cdot \chi(A_j), 1 \leq j \leq k+1$ as test functions for the Rayleigh quotient.