Math 564, Fall 2008 D. Jakobson

PROBLEM SET 3, additional problems due date TBA.

Do all the problems. Every problem is worth 5 points. Some problems will not be graded because of time constraints.

Problem 9

a) Let $N \geq n+1$ and let $A_1, A_2, \ldots, A_N$ be a finite collection of convex subsets of $\mathbf{R}^n$ such that any $(n+1)$ of them have nonempty intersection; prove that $\cap_{j=1}^{N} A_j \neq \emptyset$.

b) Prove that if we assume that A_α is convex and *compact* for all α, then the conclusion of a) holds for an arbitrary *infinite* collection $\{A_\alpha\}$.

Problem 10 Let $1/p + 1/q + 1/r = 1$, where $p, q, r > 0$. Let $f \in L^p(X, \mu), g \in L^q(X, \mu), h \in L^p(X, \mu)$. Prove that $fgh \in L^1(X, \mu)$ and that

$$||fgh||_1 \leq ||f||_p \cdot ||g||_q \cdot ||h||_r.$$

Problem 11 Let $f \in L^p(\mathbf{R})$. Show that for all $\epsilon > 0$ there exists $\delta > 0$ such that for all $|t| < \delta$, we have

$$\int_{-\infty}^{+\infty} |f(x+t) - f(x)|^p dx < \epsilon.$$