Math 480/695, D. Jakobson

Problem Set 3

Due: Thursday, February 1 in class

Additional problem 1. Let G be a k-regular graph on n vertices, and let

$$\mu \ = \ \max_{\lambda \in \sigma(A), \lambda \neq k} |\lambda|.$$

Also, let $i(G)$ denote the independence number of G. Prove that

$$i(G) \ \leq \ \frac{\mu}{k} n.$$

Hint: Let I be an independent set in G with r vertices, and let $c = r/(n-r)$. Define a function $f(x)$ on G by

$$f(x) \ = \ \begin{cases} 1, & x \in I, \\ -c, & x \notin I. \end{cases}$$

Check that f is orthogonal to the constant function, and so

$$||Af||^2 \ \leq \ \mu^2 ||f||^2$$

Also, $Af(x) = -ck$ for $x \in I$. Use this to get a lower bound for $||Af||^2$, and compare it to the upper bound in the previous formula.