Math 454, Fall 2018 D. Jakobson

PROBLEM SET 2, PART 2 **Due date to be announced**

All the problems are extra credit. Every problem are worth 10 points.

Problem 1. Every real number in $x \in [0, 1]$ can be expanded into a (finite or infinite) *continued fraction*

$$x = \cfrac{1}{n_1 + \frac{1}{n_2 + \frac{1}{n_3 + \ldots}}},$$

sometimes denoted by $x = [n_1, n_2, n_3, \ldots]$.

a) Prove that finite continued fractions correspond to rational numbers, while infinite fractions correspond to irrational numbers.

b) Let $f(x)$ denote the function $1/x - \lfloor 1/x \rfloor, x \in (0, 1)$; $f(x)$ is the fractional part of $1/x$; here $\lfloor a \rfloor$ denotes the largest integer $\leq a$). Prove that $f(x)$ can be written as a *shift map*,

$$f([n_1, n_2, n_3, \ldots]) = [n_2, n_3, \ldots].$$

c) Describe all the *periodic* continued fractions, $x = [n_1, \ldots, n_k, n_1, \ldots, n_k, \ldots]$.