McGill University

Math 355: Honors Analysis 4   Winter 2011
Topics to review for the final

- Lebesgue measure: semi-rings, rings, σ-rings; extension of a measure from a semi-ring to the minimal ring over the semi-ring.

- Additivity, σ-additivity, inner and outer measures. Lebesgue extension of a measure on a semi-ring with unity; Lebesgue measure in $\mathbf{R}^n$, translation invariance.

- Measurable functions, simple functions, limits (sum, product, max/min) of measurable functions, uniform convergence; Lusin's theorem; Egorov's theorem; convergence in measure, convergence a.e. along subsequences.

- Lebesgue integration; passage to the limit; dominated/monotone convergence; Fatou's lemma; Comparison of Lebesgue and Riemann integrals. Chebyshev inequality.

- Product measures, Fubini's theorem.

- L^p spaces, Hölder and Minkowski's inequalities.

- L^2, scalar product, parallelogram law, completeness. Convergence in the mean (i.e. in L^2). Orthogonalization, orthonormal basis, L^2 spaces with countable basis, Riesz-Fischer theorem, Bessel inequality. Hilbert spaces, linear functionals on Hilbert spaces, Parseval identity. Orthogonal subspaces, direct sum decomposition.

- Convolution: definition, convolution of 2 functions in L_1 (KF: §48, Ex. 2, p. 75; Drury: pp. 105-106; Katznelson: pp. 4-5).

- Fourier coefficients, Riemann-Lebesgue lemma, Fourier coefficients of convolutions, Fourier series (Katznelson: §1; Drury: §7).

- Good luck!