McGill University

Math 354: Honors Analysis 3 Fall 2012

Course Summary, material before midterm

Point-set topology: Topological spaces; open and closed sets; basis for topology (sufficient condition for a collection of sets to be a basis); continuous functions (criteria, equivalent definitions in metric spaces), homeomorphsims; interior, closure, boundary; limit points; neighbourhoods; metric spaces, including distance and triangle inequality; subspaces, subspace topology; product spaces, product topology; connected sets, path connected sets, their continuous images; connected components, path components; intermediate value theorem; topologist's sine curve (showing that connected does not imply path connected); compact sets, equivalent definition in $\mathbf{R}^n$; continuous image of a compact set; Hausdorff spaces; compact subset of a Hausdorff space is closed; finite and countable products of compact spaces; Tichonoff theorem; closed subsets of compact spaces; Lebesgue number of an open cover; continuous functions on compact spaces attain maximal and minimal values; totally bounded subsets of metric spaces (there exists a finite ϵ-net for every ϵ); dense sets, separable spaces.

Metric spaces: triangle inequality, normed spaces; examples: l_p, $C([0,1])$, $C^k([0,1])$; inner product spaces; Cauchy sequences, complete metric spaces, completion (definition and existence); examples of complete and non-complete metric spaces; nested ball theorem; Hölder's and Minkowski's inequalities; examples of separable and non-separable spaces; Contraction mapping theorem, applications to ODE; compact sets in l_p and in C^k; uniform convergence of continuous functions; Lipschitz functions; Arzela-Ascoli theorem, compact closure, totally bounded and uniformly equicontinuous families of functions.