McGill University

Math 354: Honors Analysis 3 | Fall 2012

Course Summary, Stein and Shakarchi

Stein and Shakarchi, Chapter 1. Almost disjoint cubes and rectangles; the Cantor set; exterior measure and its properties (monotonicity, countable sub-additivity, equal to infimum over open sets containing the set etc); Lebesgue measurable sets, Lebesgue measure and its properties; increasing/descreasing sequences of sets; symmetric difference, inner and outer regularity of the Lebesgue measure; translation invariance; approximation by F_σ and G_δ sets; non-measurable sets; step functions and simple functions; measurable functions, criteria for measurability; operations on measurable functions that preserve measurability; approximation by simple functions and step functions; Egorov's theorem (nearly uniformly convergent sequences if functions); Lusin's theorem and and nearly continuous functions; Borel-Cantelli lemma.

Stein and Shakarchi, Chapter 2. Lebesgue integral, definition and basic properties (e.g. linearity, additivity, monotonicity, triangle inequality); approximation by integrals of simple functions and step functions; Bounded Convergence theorem, Fatou's lemma, Dominated Convergence theorem; Proposition 1.12; complex-valued functions; $L^1(\mathbf{R}^d)$, Riesz-Fischer theorem; dense sets of functions in L^1; translation invariance; Theorems of Fubini and Tonnelli; product sets; convolution.