McGill University
Math 315A: Differential Equations
Assignment 7: ANSWERS

Problem 1. Solve the following differential equation using Laplace transform: $y'' + 3y' + 2y = f(t)$, $y(0) = 2, y'(0) = -1$, where $f(t)$ is the function

$$f(t) = \begin{cases} e^{-t}, & 0 \leq t < 3, \\ 1, & t \geq 3. \end{cases}$$

Answer:

$$y(t) = 2e^{-t} + te^{-t} + u(t-3) \cdot \left[-e^{3-2t} + e^{6-2t}/2 + 4e^{-t} + 1/2 - e^{3-t} - te^{-t}\right].$$

Problem 2. Find at least four nonzero terms in the power series expansion (about $x = 0$) of solutions to the equations $xy'' + (2-x)y' - y = 0$.

Answer: Roots of the indicial equation are $r = 0, -1$. Solution for $r = 0$:

$$y = 1 + x/2 + x^2/6 + x^3/24 + x^4/120 + \ldots$$

Problem 3. Use the method of Frobenius to find the first few coefficients in the power series expansion (about $x = 0$) of solutions to the following equations:

a) $3x^2y'' + 8xy' + (x-2)y = 0$.

Answer: The indicial equation has roots $r = -2, 1/3$. Solution for $r = 1/3$:

$$y = 1 - x + x^2/260 - x^3/12480 + \ldots$$

b) $xy'' + y' + y = 0$.

Answer: Indicial equation has a double toot at $r = 0$. One solution for $r = 0$:

$$y = 1 - x + x^2/4 - x^3/9 + \ldots$$

c) $x(x+1)y'' + (x+5)y' - 4y = 0$. **Answer:** Indicial equation has roots $r = -4, 0$. The solution for $r = 0$:

$$y = 1 + 4x/5 + x^2/5.$$