McGill University
Math 315A: Differential Equations
Assignment 5: ANSWERS

Problem 1. Given a differential equation and a nontrivial solution y_1, use the method of reduction of order to find another solution y_2:

1a. $xy'' + (1 - 2x)y' + (x - 1)y = 0$, $x > 0$, $y_1(x) = e^x$; Answer: $e^x \ln x$.

1b. $x^2 y'' + 6xy' + 6y = 0$, $x > 0$, $y_1(x) = x^{-2}$; Answer: $1/x^3$.

Problem 2. Find the general solution of the following differential equation:

$$x^2 y'' + 4xy' + 2y = x^3 + \ln x.$$

Answer:

$$y = a/x + b/x^2 + (x^3 + 10\ln x - 15)/20.$$

Problem 3. Find general solutions of the following equations:

3a. $y'' + 9y = \sec^2(3x)$ (variation of parameters); Answer:

$$a\cos(3x) + b\sin(3x) + \{-1 - \ln[\cos(3x/2) - \sin(3x/2)]\ \sin(3x) + \ln[\cos(3x/2) + \sin(3x/2)]\ \sin(3x)\}/9.$$

3b. $x^2 y'' + 3xy' + y = 1/x$ (Cauchy-Euler). Answer:

$$a/x + b\ln x/x + (\ln x)^2/(2x).$$

Problem 4. Find a general solution of the non-homogeneous equation

$$xy'' + (1 - 2x)y' + (x - 1)y = xe^x$$

using the two solutions y_1, y_2 of the homogeneous equation you found in Problem 1a. Answer:

$$(e^x \cdot x^2)/4 + ae^x + be^x \cdot \ln x.$$