McGill University
Math 315A: Differential Equations
Assignment 7: due Tuesday, December 2, 2003 before 5pm

1. (1pt) Solve the following differential equation using Laplace transform: $y'' + 3y' + 2y = f(t),\ y(0) = 2, y'(0) = -1$, where $f(t)$ is the function

$$f(t) = \begin{cases} e^{-t}, & 0 \le t < 3, \\ 1, & t \ge 3. \end{cases}$$

2. (1pt) Find at least four nonzero terms in the power series expansion (about $x = 0$) of solutions to the equations $xy'' + (2 - x)y' - y = 0$.

3. (3 pts) Use the method of Frobenius to find a general formula for the coefficients in the power series expansion (about $x = 0$) of solutions to the following equations:

   a) $3x^2y'' + 8xy' + (x - 2)y = 0$;

   b) $xy'' + y' + y = 0$;

   c) $x(x + 1)y'' + (x + 5)y' - 4y = 0$.

4. (1 pt) Show that $J_{1/2}(x)$ is a constant multiple of $x^{-1/2}\sin(x)$. Hint: compare the recurrence formulas.

5. (just for fun, don't turn in). Use the method of Frobenius to find the first four nonzero terms in the series expansion about $x = 0$ for a general solution of the equation $6x^3y''' + 11x^2y'' - 2xy' - (x - 2)y = 0$. Hint: let

$$y(x) = \sum_{n=0}^{\infty} a_n x^{n+r}.$$

Substitute this expression into the differential equation, and collect the terms to get the indicial equation and the recurrence formulas.