189-265A COURSE OUTLINE (September 2001)

1 Topic I. Line integrals: $2\frac{1}{2}$ weeks

1.1 Parametric curves

Review of parametrization for lines and circles. Paths and curves. Differentiation and integration of vector-valued functions. Velocity and arc length. Integration of a scalar function on a curve against arc length. Centroid and center of mass.

1. *Adams*: (third edition) §3.4, §7.1, §7.2, §8.3; (second edition) §1.4, §5.2, §6.3.
2. *Edwards & Penney* : (fifth edition) §10.4, §12.4, §12.5, §12.6, §15.2.
3. *Larson, Hostetler, Edwards*: (fifth edition) §10.2, §11.5, §12.1 – §12.5.
4. *Marsden & Tromba*: (fourth edition) §1.1, §4.1, §4.2, §7.1; (third edition) §1.1, §3.1, §3.2, §7.1.
5. *Salas & Hille*: (seventh edition) §12.5, §12.6, §13.3, §13.4; (eighth edition) §12.6, 13.1-13.5,
6. *Schaum's outline*:
7. *Stewart*: (fourth edition) §11.1, §13.5, §14.1, §14.2, §14.3, §17.2.
8. *Thomas & Finney*: (ninth edition) §10.5, §11.1, §11.3, §14.1

1.2 Vector fields

The concept of a vector field. Gradient fields. Examples. Work and flux. Conservative vector fields.

1. *Adams*: (third edition) §8.1, §8.2; (second edition) §6.1, §6.2.
2. *Edwards & Penney* : (fifth edition) §15.1, §15.2, §15.4.
3. *Larson, Hostetler, Edwards*: (fifth edition) §15.1.
4. *Marsden & Tromba*: (fourth edition) §4.3; (third edition) §3.3.
5. *Salas & Hille*: (seventh edition) §17.1.
6. *Schaum's outline*: Chapter 1.
7. *Stewart*: (fourth edition) §17.1, §17.2 (not much on work or flux).
8. *Thomas & Finney*: (ninth edition) §14.2.

1.3 Line integral I

Definition of line integral. Various formulations and notations . Basic properties and importance of orientation, in particular: $\int_{C_1+C_2} = \int_{C_1} + \int_{C_2}, \int_{-C} = -\int_C$. Independence of parametrization.

1. *Adams*: (third edition) §8.4; (second edition) §6.4.
2. *Edwards & Penney* : (fifth edition) §15.2.
3. *Larson, Hostetler, Edwards*: (fifth edition) §15.2.
4. *Marsden & Tromba*: (fourth & third edition) §7.2.
5. *Salas & Hille*: (seventh edition) §17.1.
6. *Schaum's outline*: Chapter 5.
7. *Stewart*: (fourth edition) §17.2.
8. *Thomas & Finney*: (ninth edition) §14.2

1.4 Line integral II

Review of the chain rule. $\int_C \nabla\varphi \cdot d\vec{r} = \varphi(P_1) - \varphi(P_0)$. When is a vector field conservative? The condition $\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$. The disturbing example of $\vec{F} = (\frac{-y}{x^2+y^2}, \frac{x}{x^2+y^2})$ and $\nabla \arctan(\frac{y}{x})$. Independence of path and conservativity of the vector field. Conservation of energy.

1. *Adams*:(third edition) §8.4, §8.2 Example 5; (second edition) §6.4, §6.2 Example 8.
2. *Edwards & Penney* : (fifth edition) §15.3
3. *Larson, Hostetler, Edwards*: (fifth edition) §15.3.
4. *Marsden & Tromba*: (fourth & third edition) §7.2, §8.3 Exercise 24
5. *Salas & Hille*: (seventh edition) §17.2, §17.3.
6. *Schaum's outline*: Chapter 5.
7. *Stewart*: (fourth edition) §17.3.
8. *Thomas & Finney*: (ninth edition) §14.2, §14.3.

1.5 Line integral III

The interpretation of $\int_C \vec{F} \cdot d\vec{r}$ as the flux across C of the vector field obtained from $\vec{F}$ by clockwise rotation through 90 degrees.

1. *Adams*:(second edition) §7.3 Example 6, §7.4; (third edition) §9.3 Example 6, §9.4.
2. *Edwards & Penney* : (fifth edition) §15.4
3. *Larson, Hostetler, Edwards*: (fifth edition)
4. *Marsden & Tromba*: (fourth edition) p. 475; (third edition) p. 499.
5. *Salas & Hille*: (seventh edition)
6. *Schaum's outline*:
7. *Stewart*: (fourth edition) (see §17.5 pp.1114-1115 for related material.)
8. *Thomas & Finney*: (ninth edition) §14.2

2 Topic II. Green's Theorem and double integrals: 2 weeks

2.1 Review

Double integrals. Regions of type I,II.

1. *Adams*: (third edition) §6.1; (second edition) §4.1.
2. *Edwards & Penney* : (fifth edition) §14.1, §14.2, §14.3,
3. *Larson, Hostetler, Edwards*: (fifth edition) §14.1, §14.2.
4. *Marsden & Tromba*: (fourth & third edition) §5.3.
5. *Salas & Hille*: (seventh edition) §16.2, §16.3.
6. *Schaum's outline*:
7. *Stewart*: (fourth edition) §16.3.
8. *Thomas & Finney*: (ninth edition) §13.1.

2.2 Green's theorem

Statement. Examples. Proof for a good region (i.e., simultaneously type I & II). Combination of regions. Singularities. Independence of path, simple connectivity, and $\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$. Computation of area.

1. *Adams*: (third edition) §9.4 ; (second edition) §7.4.
2. *Edwards & Penney* : (fifth edition) §15.4.
3. *Larson, Hostetler, Edwards*: (fifth edition) §15.4.
4. *Marsden & Tromba*: (fourth & third edition) §8.1.
5. *Salas & Hille*: (seventh edition) §17.6.
6. *Schaum's outline*: Chapter 6.
7. *Stewart*: (fourth edition) §17.4.
8. *Thomas & Finney*: (ninth edition) §14.

2.3 The flux form of Green's theorem

The line source $\nabla \log(x^2 + y^2)$. Applications: harmonic functions; the equation of continuity; the 2-dimensional heat equation.

1. *Adams*: (third edition) §9.3 Example 6,§9.5, Exercises §4.4 and §9.5; (second edition) §7.3 Example 6.
2. *Edwards & Penney* : (fifth edition) §15.4
3. *Larson, Hostetler, Edwards*: (fifth edition) 15.4.
4. *Marsden & Tromba*: (fourth edition) p. 475, §8.5 ; (third edition) p. 499.
5. *Salas & Hille*: (seventh edition) §17.9.
6. *Schaum's outline*:
7. *Stewart*: (fourth edition) see §17.5 (pp.1114-1115).
8. *Thomas & Finney*: (ninth edition) §14.4.

3 Topic III. Differentiation revisited: Lagrange multipliers, implicit functions: $2\frac{1}{2}$ weeks

3.1 Jacobians

Jacobian matrix (derivative) and Jacobian. Matrix chain rule (statement).

1. *Adams*: (third edition) §4.6 (second edition) §2.7.
2. *Edwards & Penney* : (fifth edition) §13.7.
3. *Larson, Hostetler, Edwards*: (fifth edition) §13.5, §14.8.
4. *Marsden & Tromba*: (fourth & third edition) §6.3. Also (fourth edition) p. 119 and (third edition) p. 125, p. 134, p. 135.
5. *Salas & Hille*: (seventh edition) §15.4.
6. *Schaum's outline*:
7. *Stewart*: (fourth edition)
8. *Thomas & Finney*: (ninth edition) §12.5, §13.7.

3.2 Change of variables in double integrals

In particular, formulas for linear transformations and polar coordinates.

1. *Adams*: (third edition) §6.4; (second edition) §4.3.
2. *Edwards & Penney* : (fifth edition) §14.4, §14.9.
3. *Larson, Hostetler, Edwards*: (fifth edition) §10.4, §14.8.
4. *Marsden & Tromba*: (fourth & third edition) §6.3.
5. *Salas & Hille*: (seventh edition) §16.11.
6. *Schaum's outline*:
7. *Stewart*: (fourth edition) §16.9
8. *Thomas & Finney*: (ninth edition) §13.7.

3.3 Critical points and the Lagrange multiplier

A brief review of critical points. The Lagrange multiplier: a curve in a level surface $F = c$ is orthogonal to ∇F; Maximizing $G(\vec{r}(t))$ when the curve lies on $F = c$.

1. *Adams*: (third edition) §5.3; (second edition) §3.3.
2. *Edwards & Penney* : (fifth edition) §13.5, §13.9
3. *Larson, Hostetler, Edwards*: (fifth edition) §13.10.
4. *Marsden & Tromba*: (fourth edition) §3.3; (third edition) §4.3.
5. *Salas & Hille*: (seventh edition) §15.8.
6. *Schaum's outline*:
7. *Stewart*: (fourth edition) §15.8.
8. *Thomas & Finney*: (ninth edition) §12.9.

3.4 Implicit differentiation

Recall of Gaussian elimination. The simple case of $F(x, y) = c$. A matrix formulation of the general case.

1. *Adams*: (third edition) §4.8; (second edition) §2.7.
2. *Edwards & Penney* : (fifth edition) §13.7
3. *Larson, Hostetler, Edwards*: (fifth edition) §13.5.
4. *Marsden & Tromba*: (fourth edition) §3.5; (third edition) §4.4.
5. *Salas & Hille*: (seventh edition) §15.4.
6. *Stewart*: (fourth edition) §15.5 (pp. 956-957).
7. *Schaum's outline*:
8. *Thomas & Finney*: (ninth edition) §12.5.

CLASS TEST

4 Topic IV. Surface integrals and Stokes' theorem: 3 weeks

4.1 Vector analysis

The basic differential operators of grad (i.e., ∇), curl (i.e., $\nabla\times$), and div (i.e., $\nabla\cdot$). The basic relation: functions $\xrightarrow{\nabla}$ vector fields $\xrightarrow{\nabla\times}$ vector fields $\xrightarrow{\nabla\cdot}$ functions. Important fact: two in succession always gives zero. Inverse square law and $\nabla \cdot \nabla(\frac{1}{r}) = 0$.

1. *Adams*: (third edition) §9.2; (second edition) §7.1.
2. *Edwards & Penney* : (fifth edition) §15.1
3. *Larson, Hostetler, Edwards*: (fifth edition) §15.1.
4. *Marsden & Tromba*: (fourth edition) §4.4; (third edition) §3.4.
5. *Salas & Hille*: (seventh edition) §17.8.
6. *Schaum's outline*: Chapter 3.
7. *Stewart*: (fourth edition) §17.1 (p. 1079), §17.5.
8. *Thomas & Finney*: (ninth edition) §12.7, §14.4, §14.7, §14.8.

4.2 Surface integration

Parametric surfaces. The standard case of the graph of a function $z = f(x, y)$. Integration of a scalar function. The flux of a vector field across a parametric surface. The case of a cylinder over a plane curve (recall flux across a curve). Example of the sphere. Orientable surfaces.

1. *Adams*: (third edition) §8.6; (second edition) §6.5.
2. *Edwards & Penney* : (fifth edition) §14.8, §15.5.
3. *Larson, Hostetler, Edwards*: (fifth edition) §15.5, §15.6.
4. *Marsden & Tromba*: (fourth & third edition) §7.3, §7.4, §7.5, §7.6.

5. *Salas & Hille*: (seventh edition) §17.6, §17.7.
6. *Schaum's outline*: Chapter 5.
7. *Stewart*: (fourth edition) §17.6, §17.7.
8. *Thomas & Finney*: (ninth edition) §14.5, §14.6.

4.3 Flux and circulation: Stokes' theorem

Statement. Examples. Gluing patches together: Stokes' theorem for an orientable surface. Simple computations over the surface of a cube (say) or a sphere. Independence of surface for $\int\int_S \operatorname{curl} \vec{G}\cdot d\vec{S}$.

1. *Adams*: (third edition) §9.4; (second edition) §7.4.
2. *Edwards & Penney* : (fifth edition) §15.5.
3. *Larson, Hostetler, Edwards*: (fifth edition) §15.6, §15.8.
4. *Marsden & Tromba*: (fourth & third edition) §8.2.
5. *Salas & Hille*: (seventh edition) §17.10.
6. *Schaum's outline*: Chapter 6.
7. *Stewart*: (fourth edition) §17.8.
8. *Thomas & Finney*: (ninth edition) §14.7.

4.4 Stokes' theorem 2

Outline of proof for a parametric surface: reduction to Green's theorem. Green's theorem is a special case. Conservative fields in $\mathbb{R}^3$.

1. *Adams*: (third edition) §9.4; (second edition) §7.4.
2. *Edwards & Penney* : (fifth edition) §15.7
3. *Larson, Hostetler, Edwards*: (fifth edition) §15.8.
4. *Marsden & Tromba*: (fourth & third edition) §8.2.
5. *Salas & Hille*: (seventh edition) §17.10
6. *Schaum's outline*: Chapter 6.
7. *Stewart*: (fourth edition) §17.8.
8. *Thomas & Finney*: (ninth edition) §14.7

5 Topic V. Triple integrals and the divergence (or Gauss) theorem: 3 weeks.

5.1 Review of triple integrals

Reduction to double integrals for the regions of type I,II, and III. The formulas for change of variables. Cylindrical and spherical coordinates.

1. *Adams*: (third edition) §6.5, §6.6; (second edition) §4.4, §4.5.
2. *Edwards & Penney* : (fifth edition) §14.6, §14.7, §14.9.

3. *Larson, Hostetler, Edwards*: (fifth edition) §14.6, §14.7.
4. *Marsden & Tromba*: (fourth & third edition) §6.1, §6.3.
5. *Salas & Hille*: (seventh edition) §16.7, §16.8, §16.9.
6. *Schaum's outline*: Chapter 5.
7. *Stewart*: (fourth edition) §16.7, §16.8.
8. *Thomas & Finney*: (ninth edition) §13.4, §13.6, §13.7.

5.2 Flux of a vector field across the boundary of a solid

Example of the sphere and curl $\vec{F}$ using Stokes' theorem and change of orientation. The general result: the divergence theorem (statement).

1. *Adams*: (third edition) §9.3; (second edition) §7.3.
2. *Edwards & Penney* : (fifth edition) §15.5, §15.6.
3. *Larson, Hostetler, Edwards*: (fifth edition) §15.7.
4. *Marsden & Tromba*: (fourth & third edition) §8.4
5. *Salas & Hille*: (seventh edition) §17.9.
6. *Schaum's outline*: Chapter 6.
7. *Stewart*: (fourth edition) §17.9.
8. *Thomas & Finney*: (ninth edition) §14.8.

5.3 The divergence theorem

Harmonic functions. Green's first identity. The flux of $\nabla\frac{1}{r}$ across a sphere. The flux of $\nabla\frac{1}{r}$ across the boundaries of other solids.

1. *Adams*: (third edition) §9.3; (second edition) §7.3.
2. *Edwards & Penney* : (fifth edition) §15.6.
3. *Larson, Hostetler, Edwards*: (fifth edition) §15.7.
4. *Marsden & Tromba*: (fourth & third edition) §8.4
5. *Salas & Hille*: (seventh edition) §17.9.
6. *Schaum's outline*:
7. *Stewart*: (fourth edition) §17.9.
8. *Thomas & Finney*: (ninth edition) §14.8.

5.4 Applications

(Optional: up to instructor) Circulation. Heat flow and the continuity equation. Magnetostatics and electric flux (see Adams). Maxwell's equations.

1. *Adams*: (third edition) §9.5 ; (second edition) §7.5.
2. *Edwards & Penney* : (fifth edition)
3. *Larson, Hostetler, Edwards*: (fifth edition)
4. *Marsden & Tromba*: (fourth & third edition) §8.5
5. *Salas & Hille*: (seventh edition)
6. *Schaum's outline*: Chapter 6.
7. *Stewart*: (fourth edition)
8. *Thomas & Finney*: (ninth edition) §14.8.