McGill University — Math 262: Intermediate Calculus, Fall 2014

WRITTEN ASSIGNMENT 3 — Due November 11, 2014

Professor: Dmitry Jakobson

INSTRUCTIONS: Answer any 5 of the following 6 questions. To get the full mark, it is not enough to state the correct answer; there should be a detailed explanation for that answer. You can use any result from the book or from the lectures, but you should explain how it applies to the problem.

Problem 1. (4 points)

Suppose $u(x, y)$ and $v(x, y)$ have continuous second partial derivatives and satisfy the *Cauchy-Riemann equations*

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \quad \textit{and} \quad \frac{\partial v}{\partial x} = -\frac{\partial u}{\partial y}.$$

Show that u and v are both harmonic (i.e. that $\partial^2 u/\partial x^2 + \partial^2 u/\partial y^2 = 0$ and $\partial^2 v/\partial x^2 + \partial^2 v/\partial y^2 = 0$).

Problem 2. (4 points)

Let $f(x, y, z)$ have continuous second partial derivatives, and let

$$x = \rho \sin\phi \cos\theta, y = \rho \sin\phi \sin\theta, z = \rho \cos\phi.$$

Define $g(\rho, \phi, \theta)$ by

$$g(\rho, \phi, \theta) = f\left(x(\rho, \phi, \theta), y(\rho, \phi, \theta), z(\rho, \phi)\right).$$

Show that

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2} =$$
$$\frac{\partial^2 g}{\partial \rho^2} + \frac{2}{\rho}\frac{\partial g}{\partial \rho} + \frac{\cot\phi}{\rho^2}\frac{\partial g}{\partial \phi} + \frac{1}{\rho^2}\frac{\partial^2 g}{\partial \phi^2} + \frac{1}{\rho^2 \sin^2\phi}\frac{\partial^2 g}{\partial \theta^2}$$

Problem 3. (4 points) Find the coordinates of all points on the following surfaces where the surface has a horizontal tangent plane.

a) $z = x^4 - 4xy^3 + 6y^2 - 2$;

b) $z = xye^{-(x^2+y^2)/2}$.

Problem 4. (4 points)

a) Let $e^y \sin x = x + xy$. Find dy/dx.

b) Let $yz + x \ln y = z^2$. Find $\partial z/\partial x$ and $\partial z/\partial y$.

Problem 5. (4 points)

a) Use Lagrange multipliers to find the maximum and minimum values of the function $f(x, y, z) = x^2 + y^2 + z^2$ subject to the constraint $x^4 + y^4 + z^4 = 1$.

b) Find the extreme values of $f(x, y, z) = x^2 + y^2 + z^2$ subject to both constraints: $x - y = 1$ and $y^2 - z^2 = 1$.

Problem 6. (4 points)

Classify the critical points of the following functions:

a) (1pt) $x^4y^4 - 4xy$;

b) (1pt) $\cos x + \cos y$;

c) (2pts) $xyz - x^2 - y^2 - z^2$.