

Extreme Jensen measures

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Abstract

Let Ω be an open subset of \mathbf{R}^d ($d \geq 2$) and let $x \in \Omega$. A Jensen measure for x on Ω is a Borel probability measure μ , supported on a compact subset of Ω , such that $\int u d\mu \leq u(x)$ for every superharmonic function u on Ω . Denote by $J_x(\Omega)$ the family of Jensen measures for x on Ω . We present two characterizations of $\text{ext}(J_x(\Omega))$, the set of extreme elements of $J_x(\Omega)$. The first is in terms of finely harmonic measures, and the second as limits of harmonic measures on decreasing sequences of domains.

This allows us to relax the local boundedness condition in a previous result of B. Cole and T. Ransford (the main result of *Jensen measures and harmonic measures*, J. Reine Angew. Math. **541** (2001), 29–53.).

As an application, we give an improvement of a result of Khabibullin on the question of whether, given a complex sequence (α_n) and a continuous function $M : \mathbf{C} \rightarrow \mathbf{R}^+$, there exists an entire function $f \not\equiv 0$ satisfying $f(\alpha_n) = 0$ for all n , and $|f(z)| \leq M(z)$ for all $z \in \mathbf{C}$.