NONEQUILIBRIUM STATISTICAL MECHANICS AND SMOOTH DYNAMICAL SYSTEMS.

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There are many different approaches to nonequilibrium statistical mechanics, involving different mathematical idealizations. The relation between these approaches remains often unclear, reflecting the fact that nonequilibrium statistical mechanics (not close to equilibrium) is currently still in a formative stage. The approach that we wish to discuss here relates classical nonequilibrium statistical mechanics with the ergodic theory of a smooth dynamical system on a compact manifold. This approach excludes quantum systems, but is otherwise physically quite reasonable. Its chief interest is that it can use the powerful techniques and results of the theory of differentiable dynamical systems (in particular hyperbolic dynamical systems). This allows one to formulate, and in part solve, problems that remain otherwise quite inaccessible.

A brief outline of the smooth dynamics approach is as follows. One considers a classical system with a finite number of degrees of freedom maintained outside of equilibrium by non gradient forces and thermostatted by deterministic forces (*e.g.*, isokinetic thermostat). The phase space is identified as a compact manifold M, and the time evolution is described by a smooth dynamical system (f^t) on M. Nonequilibrium steady states (NESS) are identified with so-called SRB measures ρ for (f^t). One argues that the rate of entropy production can be identified with the rate of volume contraction in M with respect to (f^t). Assuming that (f^t) is uniformly hyperbolic (chaotic hypothesis) gives then in particular the Gallavotti-Cohen Fluctuation Theorem.

Of particular interest is the theory of linear response: how does the NESS ρ depend on (f^t)? A very satisfactory answer is obtained in the case of uniformly hyperbolic dynamics. One can define a susceptibility function analytic in the upper half-plane (this expresses causality), and one obtains – far from equilibrium – a modified version of the Fluctuation-Dissipation Theorem known to hold close to equilibrium. When uniform hyperbolicity is not satisfied one can have singularities of the susceptibility in the upper half-plane (in apparent violation of causality). In particular, such singularities of the susceptibility are known to be present in the case of the Hénon attractor. The physical significance of these acausal singularities remains to be analyzed. Perhaps they are not visible in situations of physical interest for nonequilibrium statistical mechanics.