Given (M,g) a compact Riemmanian manifold of dimension $n>2$, we are interested in the existence of blowing-up sign-changing families $(u_\epsilon)_{\epsilon>0}\in C^{2,\theta}(M)$, $\theta\in(0,1)$, of solutions to

$$\Delta_g u_\epsilon + h u_\epsilon = |u_\epsilon|^{\frac{4}{n-2}-\epsilon} u_\epsilon \text{ in } M$$

where $\Delta_g := -\mathrm{div}_g(\nabla)$ and $h\in C^{0,theta}(M)$ is a potential. We prove that such families exist in two main cases: in small dimension $n\in\{3,4,5,6\}$ for any potential h or in any dimension when $h\equiv\frac{n-2}{4(n-1)}Scal_g$ and (M,g) is locally conformally flat. These examples complete previous existence and nonexistence results on blowing-up solutions and allow to have a complete panorama of the stability/instability of critical elliptic equations of scalar curvature type on compact manifolds. The changing of the sign is necessary due to the compactness results of Druet and Schoen.