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A new approach to the L^p-theory of $-\Delta+b\cdot\nabla$, and its application to Feller processes with general drifts.

The problem of constructing a Feller process on $\mathbb{R}^d$, $d \geqslant 3$, having infinitesimal generator $-\Delta+b\cdot\nabla$, with a singular vector field $b:\mathbb{R}^d \to \mathbb{R}^d$ ("a diffusion with drift b") has been thoroughly studied in the literature, motivated by applications to Mathematical Physics, as well as the search for the maximal (general) class of vector fields b such that the associated process exists. This search culminated in several distinct classes of singular drifts (including $L^d + L^\infty$, the best possible result in terms of L^p spaces). I construct a process with b in a wide class of vector fields (measures), containing the classes previously known, and combining, for the first time, critical point singularities and critical hypersurface singularities.

I introduce a new method "of constructing the resolvent": the starting object is an operator-valued function, a 'candidate' for the resolvent of an operator realization $\Lambda_p(b)$ of $-\Delta + b\cdot\nabla$ generating a holomorphic C_0-semigroup in L^p (the key observation here is a link between $-\Delta + b\cdot\nabla$ and $(\lambda-\Delta)^{-\frac{1}{2}} + |b|$, as opposed to the classical approach, which compares $-\Delta + b\cdot\nabla$ to $-\Delta + b^2$). The very form of this function provides a detailed information about smoothness of the *domain* $D(\Lambda_p(b))$. Now, this information about $D(\Lambda_p(b))$, combined with the Sobolev embedding theorem, allows us to move the burden of the proof of convergence in the space C_∞ of continuous functions on $\mathbb{R}^d$ vanishing at ∞ (as needed to construct the transition probability function of the process) to L^p, a space having much weaker topology (locally), hence the gain in the admissible singularities of the drift.

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